

The complex nonlinear systems with fractal as well as chaotic dynamics of annual runoff processes in the three headwaters of the Tarim River

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Abstract: This paper attempted to identify fractal and chaotic characteristics of the annual runoff processes in headwaters of the Tarim River. Methods of fractal analyses were used to explore several aspects of the temporal changes from 1957 to 2002. The main findings are as follows: (1) The annual runoff processes of the three headwaters of the Tarim River are complex nonlinear systems with fractal as well as chaotic dynamics. (2) The correlation dimensions of attractor derived from the time series of the annual runoff for the Hotan, Yarkand and Aksu rivers are all greater than 3.0 and non-integral, implying that all three rivers are chaotic dynamical systems that are sensitive to initial conditions, and the dynamic modeling of their annual runoff process requires at least four independent variables. (3) The time series of annual runoff in each river presents a long-term correlation characteristic. The Hurst exponent for the period of 1989 to 2002 suggests that we may expect to see an increasing trend in the annual runoff of the Aksu and Yarkand rivers in the years after 2002, but a decreasing tendency for the Hotan River in the same period.

Keywords: annual runoff process; headwater; Tarim River Basin; correlation dimension; Hurst exponent

1 Introduction

Water shortage and related ecological degradation are the most pressing environmental issues in North and Northwest China (Fang *et al.*, 2007; Xia *et al.*, 2007). This region contains China's largest continental river basin, i.e. the Tarim River Basin, with rich natural resources, yet presents an extremely high vulnerability in sustaining its ecosystem. As a most critical ecological element yet a least available resource, water is both loved and hated at the same time among the local residents on this arid and semi-arid land. The distribution and usage of water there have become a simple matter that whether to develop the economy or to protect

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the ecosystem, or to balance both. This situation has been severely restricting the sustainable development of the regional economy in the Tarim River Basin (Chen *et al.*, 2004; Liu and Chen, 2007).

This paper concerns a fundamental question about the carrying capacity of the ecosystem in the Tarim River Basin, i.e. how much water is really available in the basin for human sustainable development. Although the majority of water in the Tarim River comes from its three headwaters, the water budget in the basin is a rather complex equation. It consists of the runoff of each headwater fed directly by the glaciers and snowmelt from the surrounding mountains, all elements in the natural evapo-transpiration process, as well as human consumption. For this reason, more and more research attention has been paid to the runoff processes and their hydrological response to climate change in the three headwaters of the Tarim River (Chen *et al.*, 2005, 2006; Jiang, 2007).

It is widely recognized that a river channel runoff process is a complex system, which has been characterized as nonlinear (Strupezewski *et al.*, 2006; Liu *et al.*, 2006; Sivakurnar, 2007; Wang *et al.*, 2008). Many researchers have explored the nonlinearity of runoff processes using various nonlinear analytic methods and models, including wavelet, artificial neural networks, fractal theory (Wilcox *et al.*, 1991; Smith *et al.*, 1998; Chou, 2007; Hu, 2008; Movahed and Hermanis, 2008) and statistical method (Lin *et al.*, 2008; Chen H *et al.*, 2007; Xu, 2002). However, it has proven difficult to achieve a thorough understanding of the nonlinear mechanism of a particular runoff process. As to the three headwaters of the Tarim River, we are still far from having an in-depth understanding of their nonlinear runoff characteristics, and this forms the focus of this paper. In our study, we attempted to reveal their chaotic dynamical characteristics with a correlation dimension analysis, and their long-term correlation characteristics by using the R/S analysis method.

2 Area, data and methods

2.1 Study area

The Tarim River Basin, with an area of 1.02×10^6 km², is the largest continental river basin in China. It covers the entire southern part of Xinjiang in western China that is characterized as with both rich natural resources and fragile environment. This region has an extreme desert climate with an average annual temperature of 10.6–11.5°C. The monthly mean temperature ranges from 20 to 30°C in July and –10 to –20°C in January. The highest and lowest temperatures are +43.6°C and –27.5°C respectively. The accumulative temperature > 10°C ranges from 4100 to 4300°C. The annual precipitation is about 116.8 mm for the entire area. The figure ranges from 200 to 500 mm in the mountainous area, 50 to 80 mm on the edges of the basin, and only 17.4–25.0 mm in the central area of the basin. There is great temporal unevenness in precipitation within any year. More than 80% of the total annual precipitation falls between May and September in the high-flow season, and less than 20% of the total falls from November to April.

The main channel of the Tarim River is 1321 km in length. Naturally and historically the Tarim River Basin consists of 114 rivers from nine drainage systems, which include Aksu, Hotan, Yarkand, Qarqan, Keriya, Dina, Kaxgar, Kaidu–Konqi Rivers. The basin covers an

arable land area of 20.44×10^6 ha and has a human population of 8.26×10^6 . The mean annual natural surface runoff is $3.98 \times 10^{10} \text{ m}^3$, which originates mostly from glaciers, snowmelt and precipitation in the surrounding mountains.

Intensive disturbances caused by human activities, particularly excessive water resources exploitation, have brought about marked changes during the past 50 years. The drainage systems gradually disintegrated when the Weigan, Kaxgar, Dina, Keriya and Qarqan Rivers stopped flowing to the mainstream and were eventually disconnected from it. Today, there are only three drainage systems connected to the mainstream of the Tarim River. These are the Aksu River, Yarkand River and Hotan River. With two main sub-streams (i.e., Tongshigan and Kumalak rivers), the Aksu River originates from the Tianshan Mountains in the northwest of the basin. The Hotan River, also having two main sub-streams (i.e. the Kalakash and Yulongkash rivers), starts from the Kunlun Mountains and flows over the southwestern part of the basin. The Yarkand River originates from the Pamir Plateau and lies between the above two rivers (Figure 1).

As mentioned before, glaciers, snowmelt and precipitation in the surrounding mountains are the source of runoff for the Tarim River. Glacial/snow melt water makes up 48.2% of the total runoff. Interannual runoff variability is small, with a coefficient of variation ranging from 0.15 to 0.25 and the maximum and minimum modular coefficients are 1.36 and 0.79 respectively. Seasonal runoff is unevenly distributed. Runoff in the June–August flood season accounts for 60%–80% of the annual total (Chen *et al.*, 2003).

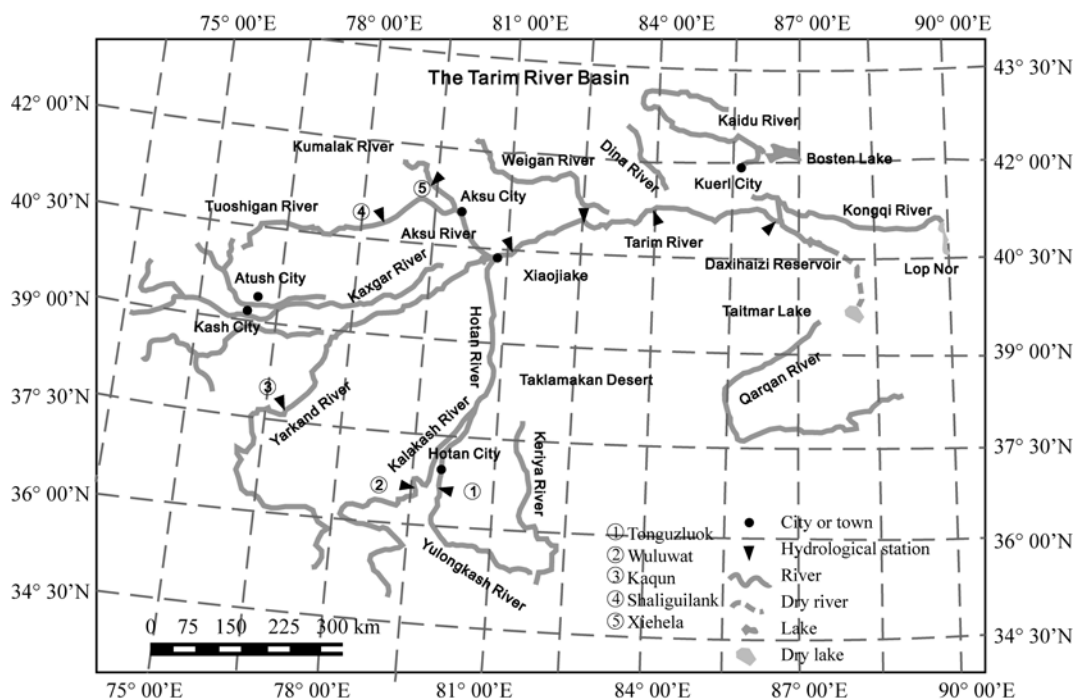


Figure 1 The sketch map of the Tarim River Basin

2.2 Data

To analyze the long-term trend of annual runoff in the three headwaters of the Tarim River, the runoff data from 1957 to 2002 in the five sub-rivers were used. The data for the Aksu River was obtained from the Xiehela and Shaliguilank hydrologic stations, the Yarkand River from the Kaqun hydrologic station, and the Hotan River from the Tonguzluok and Wuluwat hydrologic stations, respectively. Since all the stations are located in the source areas of these rivers, and the amount of water used by human within every sub-river basin is relatively small compared to the total discharge, we assumed that the monitoring hydrological data reflect the natural condition.

Long-term climate change can alter the runoff production pattern, the time of hydrological events and the frequency and severity of floods, particularly in arid and semi-arid regions; therefore, a small change in precipitation and temperature may result in significant changes in runoff (Gan, 2000). For such reasons, the data on the annual runoff in the three headwaters of the Tarim River from 1957 to 2002 were used in this study.

2.3 Methods

All statistical methods analysis are based on the same assumption that all data of time series are independent (i.e., conforming to the Gauss distribution), hence the series is stochastic. When H E Hurst (1951, 1965), a British physicist, analyzed water level of the Nile River, he found that time-series variables such as river water level did not exhibit this distributional characteristic. Instead, it showed the characteristics of discontinuity and durability. Based on the empirical findings of Hurst, B B Mandelbrot (1968, 1973) made a breakthrough regarding fundamental theories of traditional statistical methods. He found many time series no longer present a random Brownian movement unrelated to the past, but show a characteristic of long-term correlation (Comte *et al.*, 1996), which he called “fractal”.

To achieve a better understanding of the nonlinear characteristics of the annual runoffs in the three headwaters of the Tarim River, we employed a two-step comprehensive analysis method in this study. In a logical sequence, a correlation dimension method was used to show their chaotic dynamical characteristics, and then an R/S analysis method was employed to study the long-term correlation characteristics.

2.3.1 Correlation dimension

Correlation dimension method is usually applied to analyze the runoff series and check whether the river flow exhibits a chaotic dynamic characteristic or not (Sivakumar, 2007; Xu *et al.*, 2008). Its principle is as follows. Consider $X(t)$, the time series of annual runoff, and suppose it is generated by a nonlinear dynamic system with m degrees of freedom. In order to restore the dynamic characteristic of the original system, it is necessary to construct an appropriate series of state vectors $X^{(m)}(t)$ with delay coordinates in the m -dimensional phase space according to the basic ideas initiated by Grassberger and Procaccia (1983):

$$X^{(m)}(t) = \{X(t), X(t+\tau), \dots, X(t+(m-1)\tau)\} \quad (1)$$

where m is called the embedding dimension and τ is an appropriate time delay.

The trajectory in the phase space is defined as a sequence of m dimensional vectors. If the dynamics of the system is reducible to a set of deterministic laws, the trajectories of the system converge towards the subset of the phase space, which is called an “attractor”. Many

natural systems do not conform with time to a cyclic trajectory. Some nonlinear dissipative dynamic systems tend towards the attractors on which the motion is chaotic, i.e. not periodic and unpredictable over long times. The attractors of such systems are called strange attractors.

For the set $\{X_i | i = 1, 2, \dots, N\}$ of points on the attractor, using the G-P method (Grassberger and Procaccia, 1983), the correlation-integrals is defined in order to distinguish between stochastic and chaotic behaviors:

$$C(r) = \lim_{N \rightarrow \infty} \frac{1}{N^2} \{ \text{Number of pairs } (i, j), \text{ whose distance } |X_i - X_j| < r \} \quad (2)$$

where r is surveyor's rod for distance.

The correlation-integrals is computed as follows:

$$C(r) = \frac{1}{N_R^2} \sum_{j=1}^{N_R} \sum_{i=1}^{N_R} \Theta(r - |X_i - X_j|) \quad (3)$$

where N_R is the number of reference points taken from N , and N is the number of points $X^{(m)}(t)$. The relationship between N and N_R is $N_R = N - (m-1)\tau$. $\Theta(x)$ is the Heaviside function defined as

$$\Theta(x) = \begin{cases} 0 & x \leq 0 \\ 1 & x > 0 \end{cases} \quad (4)$$

The expression counts the number of points out of the data set, which are closer than radius r or within a hypersphere of radius r , and then divides it by the square of the total number of points (because of normalization). As $r \rightarrow 0$, the correlation exponent d is defined as:

$$C(r) \propto r^d \quad (5)$$

Apparently, the correlation exponent d is thus given by the slope coefficient of $\log C(r)$ versus $\log r$. According to $(\log r, \log C(r))$, d can be obtained by the least squares method (LSM) in a log-log grid.

For detecting the chaotic behavior of the system, the correlation exponent has to be plotted as a function of the embedding dimension (as Figure 2b). If $X^{(m)}(t)$ is purely random (e.g. white noise), the correlation exponent increases with the embedding dimension without reaching the saturation value.

If there is deterministic dynamics in the system, the correlation exponent reaches the saturation value, which means that it remains approximately constant with an increase of the embedding dimension. The saturated correlation exponent is called the correlation dimension of the attractor. The correlation dimension belongs to the invariants of the motion on the attractor. It is generally assumed that it equals the number of degrees of freedom of the system and higher embedding dimensions are therefore redundant. For example, in order to describe the position of the point on the plane (two-dimensional system), the third dimension is not necessary – it is redundant. The correlation dimension is often fractal: it is the non-integral dimension, which is typical for the chaotic dynamical systems that are very sensitive to initial conditions.

The correlation dimension provides information on the dimension of the phase-space required for embedding the attractor. It is important for determining the number of dimensions necessary to embed the attractor and the number of variables present in evolution of the

process.

2.3.2 R/S analysis method

R/S analysis is also called rescaled range analysis, which is usually applied to analyze long-term records of runoff time series (Zhou *et al.*, 2006; Xu *et al.*, 2008). Its principle is as follows. Considering the time series of annual runoff in a certain river, $X(t)$, for any positive integer $\tau \geq 1$, the mean value series is defined as

$$\langle X \rangle_{\tau} = \frac{1}{\tau} \sum_{t=1}^{\tau} X(t) \quad \tau = 1, 2, \dots \quad (6)$$

The accumulative deviation is

$$X(t, \tau) = \sum_{u=1}^t (X(u) - \langle X \rangle_{\tau}) \quad 1 \leq t \leq \tau \quad (7)$$

The extreme deviation is

$$R(\tau) = \max_{1 \leq t \leq \tau} X(t, \tau) - \min_{1 \leq t \leq \tau} X(t, \tau) \quad \tau = 1, 2, \dots \quad (8)$$

The standard deviation is

$$S(\tau) = \left[\frac{1}{\tau} \sum_{t=1}^{\tau} (X(t) - \langle X \rangle_{\tau})^2 \right]^{\frac{1}{2}} \quad \tau = 1, 2, \dots \quad (9)$$

When analyzing the statistic rule of $R(\tau)/S(\tau) \triangleq R/S$, H E Hurst discovered a relational expression

$$R/S \propto \left(\frac{\tau}{2}\right)^H, \quad (10)$$

which can be used to identify Hurst phenomenon in the time series, and where H is called the Hurst exponent. Apparently, H is given by the slope coefficient of R/S versus $\tau/2$. According to $(\tau, R/S)$, H can be obtained by least squares method (LSM) in a log-log grid.

Hurst *et al.* (1965) once proved that if $\{X(t)\}$ is an independently random series with limited variance, the exponent $H=0.5$; and H ($0 < H < 1$) is dependent on an incidence function $C(t)$:

$$C(t) = 2^{2H-1} - 1 \quad (11)$$

When $H > 0.5$, $C(t) > 0$; it means that the process has a long-enduring characteristic, and the future trend of the time series will be consistent with the past. In other words, if the past showed an increasing trend, the future will also show an increasing trend. When $H < 0.5$, $C(t) < 0$; it means that the process has an anti-persistence characteristic, and the future trend of the time series will be opposite from the past. In other words, if the past showed an increasing trend, the future will assume the reducing trend. When $H = 0.5$, $C(t) = 0$, indicating that the process is stochastic. In other words, there is no correlation or only a short-range correlation in the process (Ai and Li, 1993; Xu *et al.*, 2004).

3 Results and discussion

3.1 The chaotic dynamical characteristic

In Figure 2, we used the Hotan River's time series of annual runoff to reconstruct the phase space and calculated the correlation dimension of attractor. Figure 2a shows the relationship

between $\ln C(t)$ and $\ln(r)$ for the annual runoff in the Hotan River with different embedding dimension m . Using the least square method (LSM), we calculated the slope coefficient of $\ln C(r)$ versus $\log r$, i.e., the correlation exponent for embedding dimension $m=1, 2, \dots$. Figure 2b reveals the gradual saturation process of the correlation exponent. It is evident that the correlation exponent increases with embedding dimension m , and a saturated correlation exponent, i.e., the correlation dimension of attractor (D) was obtained when $m \geq 6$.

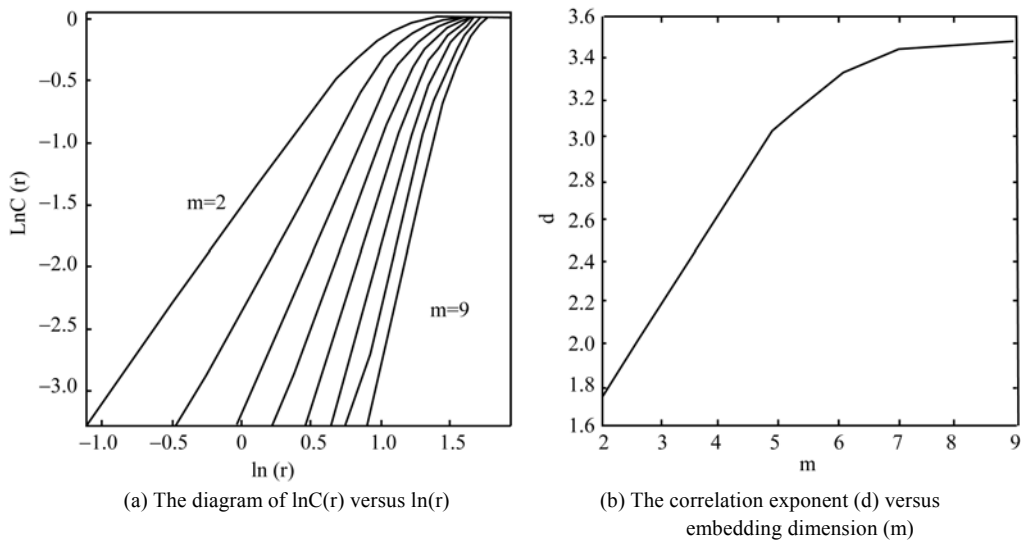


Figure 2 The wavelet approximations for annual runoff in the Hotan River at different time scales

Through the above procedure, we obtained the correlation dimension of attractor for the Yarkand River and Aksu River. Thus the correlation dimension of attractor for the Hotan, Yarkand and Aksu rivers is 3.2227, 3.2118 and 3.2092, respectively (Table 1). The fact that none of the correlation dimensions is an integer indicates that the annual runoff processes in all three headwaters are chaotic dynamical systems and very sensitive to initial conditions. Since the value of the index is above 3 for all three headwaters, at least four independent variables are needed to describe the dynamics of annual runoff process in each river.

Table 1 The correlation dimensions for the annual runoff processes in the three headwaters of the Tarim River

Rivers	Hotan River	Yarkand River	Aksu River
Embedding dimension (m)	7	7	7
Attractor dimension (D)	3.2227	3.2118	3.2092

3.2 The long-term correlation characteristic

Our studies (Xu *et al.*, 2008) revealed long-term temporal trends for the annual runoff of the Tarim River at the time scale of 16 years. During the past decades, the annual runoff tended to increase in the Aksu River and the Yarkand River but decrease in the Hotan River (Chen YN *et al.*, 2006). If this time frame is divided into three periods with roughly the same range

of 16 (2^4) years, i.e., 1957–1972, 1973–1988 and 1989–2002, the annual runoff tendency in the Aksu River, the Yarkand River and the Hotan River during each period can be clearly shown in Figures 3–5, in which the solid line represents the time series of annual runoff, and the broken line represents its linear trend.

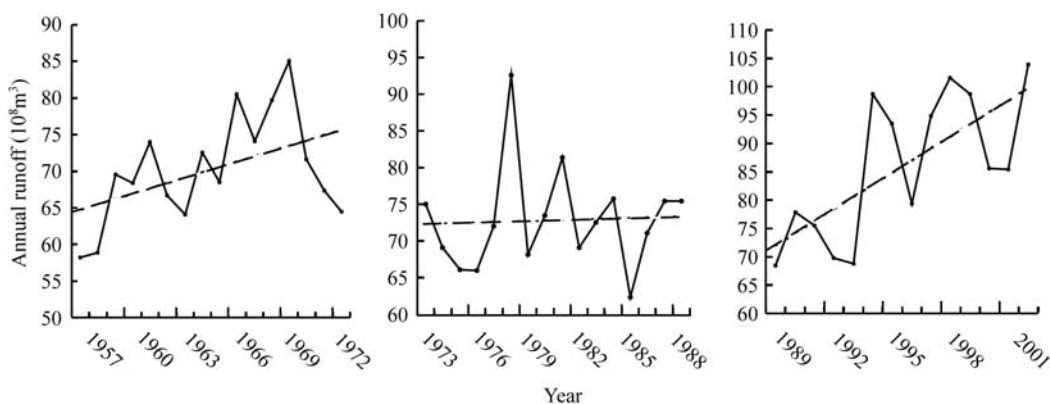


Figure 3 The annual runoff trends of the Aksu River in different periods

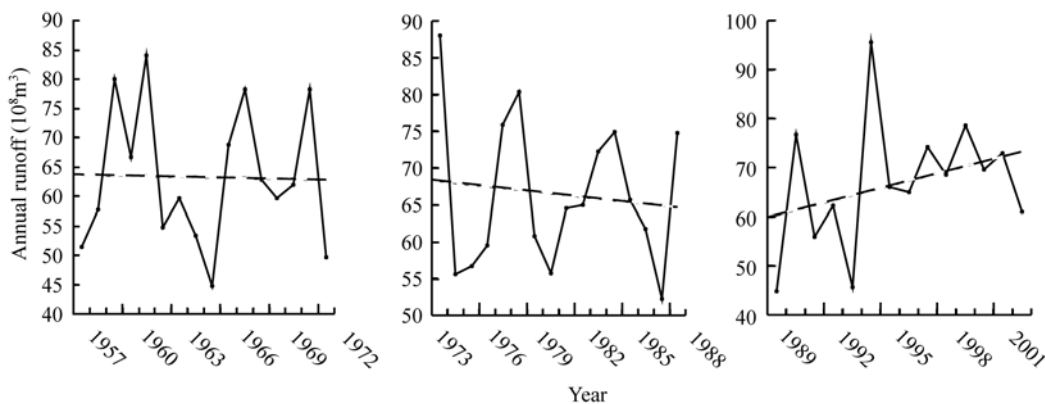


Figure 4 The annual runoff trends of the Yarkand River in different periods

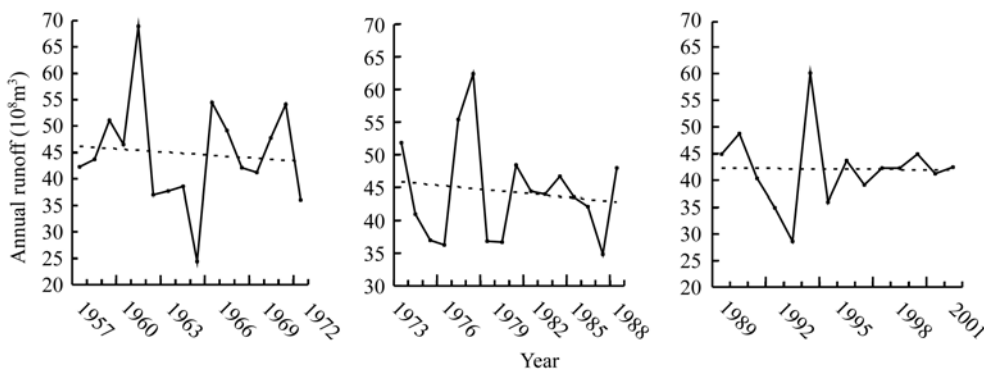


Figure 5 The annual runoff trends of the Hotan River in different periods

Using the R/S analysis method, we calculated the Hurst exponents for annual runoff in each river during each period, as shown in Table 2.

Table 2 Hurst exponents for the annual runoff processes in the three headwaters of the Tarim River

River Name	1957–1972	1973–1988	1989–2002
Aksu River	0.7632	0.5984	0.9083
Yarkand River	0.6425	0.4696	0.7342
Hotan River	0.7354	0.6415	0.5214

For the Aksu River, the Hurst exponents in the three periods, 1957–1972, 1973–1988 and 1989–2002 are 0.7632, 0.5984 and 0.9083 respectively. Since they are all greater than 0.50, it is suggested that the annual runoff process in this river has a long-enduring characteristic in every period. That is to say, the next period will have the same trend as the preceding period. The trends correspond with the conclusions from other researches (Chen YN *et al.*, 2006; Chen Y *et al.*, 2006). This characteristic is illustrated in Figure 3, where a pattern of incremental change is evident when moving from the first period to the third one. These trends correspond to the results indicated by the Hurst exponents. Also since the Hurst exponent for the last period (i.e. 1989 to 2002) is greater than 0.50, we can safely project that the annual runoff of the Aksu River may probably increase over another 14 years after 2002.

For the Yarkand River, the Hurst exponent for the period of 1957–1972 equals 0.6425, also indicating an enduring characteristic for the runoff. In the second period (1973–1988), however, the Hurst exponent (with a value of 0.4696) is less than 0.50, suggesting an anti-persistence characteristic for the annual runoff process. In the last period (1989–2002), the Hurst exponent (with a value of 0.7342) is greater than 0.50, implying an enduring characteristic again for the annual runoff process. This pattern is clearly depicted in Figure 4, with the annual runoff decreasing slightly in the first two periods and then picking up in the last period. The trends correspond with the results of Chen YN *et al.* (2006) and Zhen *et al.* (1998). Again, these trends are consistent with the results indicated by the Hurst exponents. Since the Hurst exponent for the last period is greater than 0.50, we can predict, with a considerable confidence, an ascending trend of the annual runoff for the Yarkand River over the next 14 years after 2002.

The Hurst exponents in the three periods for the Hotan River are 0.7354, 0.6415 and 0.5214 respectively. Because of all greater than 0.50, a long-enduring characteristic is implied for the annual runoff process of every period. Referring to Figure 5, where the ascending trend of annual runoff in all the three periods is clearly shown, we may also conclude the comparability between the calculated Hurst exponents and the runoff trends for this river. The trends correspond to the study results from other scholars (Zhang *et al.*, 2007; Chen YN *et al.*, 2006). The fact that the Hurst exponent for the last period is slightly greater than 0.5 indicates that, the annual runoff of the Hotan River is likely to show a slight decrease over the next 14 years after 2002.

4 Conclusions

Based on the analytic results, interpretation, and discussions presented, we may come to the

following conclusions for our study:

(1) The annual runoff processes of the three headwaters of the Tarim River are complex nonlinear systems with fractal as well as chaotic dynamics.

(2) The correlation dimension of attractor for the annual runoff process in the Hotan, Yarkand and Aksu rivers is respectively 3.2227, 3.2118 and 3.2092, and none of them is integral. This means that the annual runoff processes in all three headwaters of the Tarim River are chaotic dynamical systems, and they are very sensitive to initial conditions. Since all three calculated correlation dimensions are above 3, at least four independent variables are required to describe the dynamics of annual runoff process in each river.

(3) The time series of annual runoff in the three headwaters of the Tarim River present a long-term correlation characteristic. The Hurst exponents in the period from 1989 to 2002 indicates that the annual runoff in the Aksu and Yarkand rivers will probably show an increasing trend, but that in the Hotan river is likely to show a slightly decreasing trend in the years after 2002.

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