Grey modelling the groundwater level dynamic in the lower reaches of Tarim River affected by water delivery from upper reaches: A demonstration from Yingsu section

Jianhua Xu, Yaning Chen, and Weihong Li

Abstract—**Using the grey system theory and the monitored data from the monitoring section of Yingsu, this paper models the groundwater level dynamic in the lower reaches of Tarim River affected by water delivery from upper reaches. The main conclusions are: (1) Discharging volume, running days for water delivery and daily discharging volume, which related with water delivery from the upper reaches of Tarim River, are three main factors that markedly control and affect the groundwater level. (2) The sensitivity of groundwater level changing respond to itself becomes more and more lower versus the distance apart from river center, and the affection from discharging volume and running days for water delivery to the change rate of groundwater level becomes more and more significant with increase of the distance apart from river center. Water delivery not only markedly controls and raises the groundwater level near river, but also affects the groundwater level as far as the range in the distance of 1050 m apart from river center.**

Key words—**groundwater level; dynamic; the lower reaches of Tarim River; water delivery; grey system, modelling**

I. INTRODUCTION

The 1321-kilometer Tarim River runs west to east along the northern edge of the Taklimakan Desert, China's largest, and flows into Taitema Lake. The river is the most important source of water in semi-arid Xinjiang, with more than 8 million people living in oases clustered along its banks and in an alluvial plain downstream. Yet, with the increasing population, due to excessive water use for irrigation, industrial and living consumption in the upper and middle reaches of Tarim River, its downstream, which extends further down from the Daxihaizi Reservoir, has become completely dry ever since 1972. As a consequence, the ground-water depth in this area has lowered sharply and the ecosystem is damaged seriously. Subsequently, many plant communities gradually diminish or completely die away in a turn of grass, shrub and arbor [1]; the situation of desertification of this region becomes more and more aggravated [2]. With a full length of 428 km, the lower reaches of Tarim River are located in the eastern part of Tarim Basin in Xinjiang. Surrounded by Taklimakan desert and Kuluk desert, as one of the most drought regions in the western part of China, the lower reaches of Tarim River have serious ecological and environmental problems.

In order to prevent desertification, protect and rehabilitate the ecosystem in the lower reaches of Tarim River, China central government launched an emergency water delivery program in 2000. So far, there have been 11 continuous-time water delivery processes from Daxihaizi Reservoir to the lower reaches of Tarim River since May 2000. Based on the previous three water delivery processes, 4th water delivery process made water flowing into Taitema Lake where is in the lower reaches of Tarim River On 2 November 2001, which was the first time for water moistening the dried land on the bottom of the lake since it had dried for 30 years.

The dynamic of groundwater level in the lower reaches of Tarim River is more complicated, which is controlled and affected by not only various nature process but also various human activities. Especially, water delivery project from the upper reaches of Tarim River has been an assignable cause since 2000, which also controlled and affected the groundwater in lower reaches [3-4]. Though research results show that groundwater level in the lower reaches of Tarim River has went up obviously under the effect of water delivery [4], but because of lack in effective methods and sufficient details, modelling studies on dynamic of the groundwater level affected by transported water from upper reaches are insufficient so far [11] [14]. In fact, from the point of view of grey system theory [5]-[10] [13], the dynamic of groundwater level in the lower reaches of Tarim River affected by many complex interacting factors which include water delivery from upper reaches is typical grey system, which maybe provides us one of methods to approach the problem [15].

Maybe to figure down the dynamic and influence factors of groundwater level in all lower reaches of the Tarim River based on limited information and observed data is difficult. As an attempt, this paper use the grey system theory and

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choose the monitoring section, Yingsu, which is 45 km away from Daxihaizi Reservoir along Tarim River and the running water for every times water delivery crossed here, as focal to analyze and model the dynamic of groundwater level in the lower reaches of Tarim River affected by water delivery from upper reaches.

II. STUDY AREA AND DATA

A. *Study Area*

The overall length of the lower reaches of Tarim River from Qiala to Taitema Lake is 428 km; the channel bed stretches from north to south on alluvial fans, which is located between Taklamakan Desert and Kuluke Desert (Fig. 1). The ground surface is relatively flat, the elevation decreasing from north to south. Water seeps from streams into the alluvial fans, which can recharge shallow aquifers. The region of the lower reaches of Tarim River is classified as arid warm temperate zone, the annual precipitation varies in the range 17.4 ~ 42.0 mm, and the total annual potential evaporation is approximately 2500~3000 mm. Though in arid area and the climate with frequently strong wind-blown sand, the lower reaches of Tarim River possessed the good reputation of “Green Corridor” in history because of the well-growing vegetation on the riversides. The flora consists of 14 families, 24 genera, and about 40 species of vascular plants [11]. Riverbank vegetation provides a natural defense against the wind by obstructing sand movement, and the famous "Green Corridor" plays an important role in keeping National Highway 218 free of obstructions.

In the past 50 years, because of the excessive exploitation and unreasonable utilization of water resources in the upper reaches of Tarim River basin, the channel flow in a 320-km section in the lower reaches has been cut off since 1970’s, which caused drying-up of streams and lakes at the tail of the river. As a consecutive result, the river course broke and groundwater level dropped down greatly, a large area of Populus euphratica forest relying on groundwater for their survival and growth have died and most of shrubs and herbs become into fading, which results in the land desertification aggravating and the ecosystem was damaged seriously. In order to prevent desertification, protect and rehabilitate the ecosystem in the lower reaches of Tarim River, China central government launched an emergency water delivery project in May 2000, transporting water along river channel from up reaches lower reaches. From 2000 to 2006, there have been 11 continuous-time water delivery processes from Daxihaizi Reservoir to lower reaches of Tarim River finished.

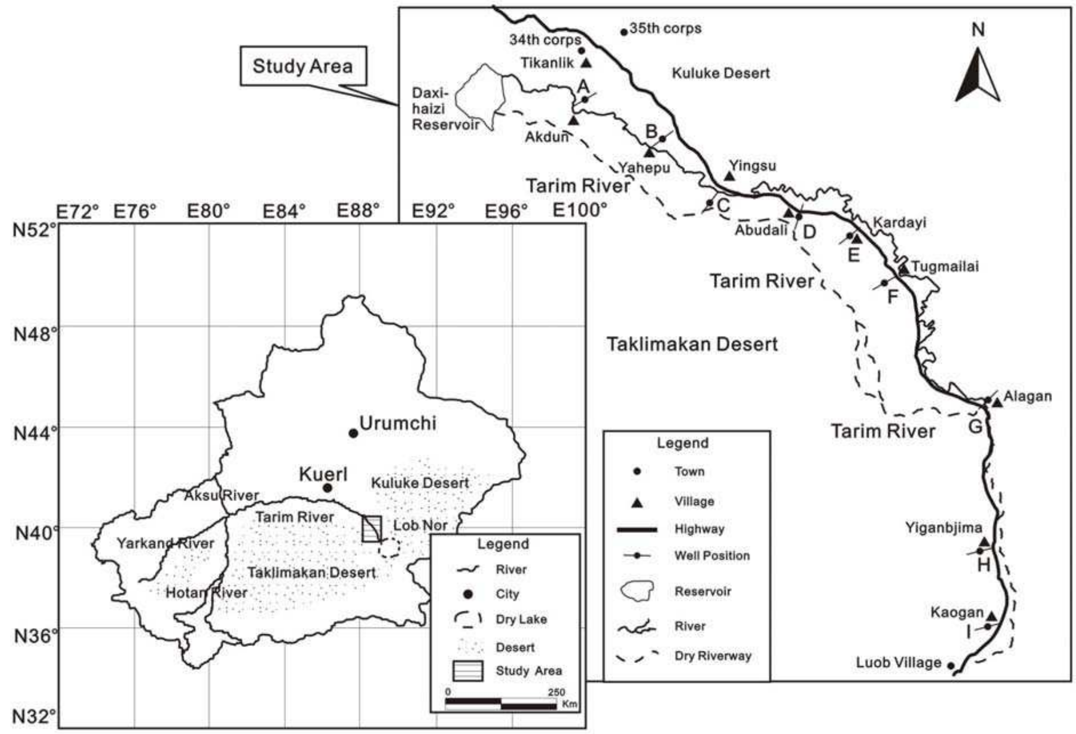

Fig. 1. Locations of the nine monitoring section along Tarim River in the lower reaches

B. Data

In order to survey the dynamic of groundwater in the lower reaches of Tarim River as a whole, nine monitoring sections along Tarim River were set up, which are Akdun (section A), Yahepu (section B), Yingsu (section C), Abudali (section D), Kardayi (section E), Tugmailai (section F), Alagan (section G), Yganbjima (section H) and Kaogan (section I). The locations of each monitoring section are showed in Figure 1. There are several monitoring wells on each monitoring section designed at different locations apart from the river center. The monitored data of groundwater level were obtained by electric conductance method non-periodically from each monitoring well. Generally, the monitored interval is 5~10 days during the period in water delivery process, and 15~30 days during the period out water delivery process.

Among of the nine monitoring sections, the section C, i.e. Yingsu is the third monitoring section that is 45 km away from Daxihaizi Reservoir and the running water for every times water delivery crossed here. So this paper chose section C as a focal section, where the groundwater level dynamic affected by the delivering water from the upper reaches of Tarim River was studied. There are eight monitoring wells on the monitoring section of Yingsu (i.e. section C), and the elevation, latitude, longitude, and the distance apart from river center of each monitoring well are showed in Table I.

TABLE I
ELEVATION, LATITUDE, LONGITUDE, AND DISTANCE APART FROM RIVER CENTER OF EACH MONITORING WELL ON THE MONITORING SECTION OF YINGSU

Well	Elevation (m)	Latitude	Longitude	The Distance apart from the river center （m）
C1	841	N40°26.123'	E87° 56.535'	230
C2	839	N40° 25.921'	E87° 56.459'	50
C3	845	N40° 5.918'	E87° 56.458'	150
C4	845	N40°25.869'	E87° 56.437'	250
C5	852	N40° 5.817'	E87° 56.418'	350
C6	850	N40° 5.766'	E87° 56.398'	450
C7	849	N40°25.646'	E87° 56.354'	750
C8	850	N40° 5.532'	E87° 56.315	1050

Because of lower elevation, monitoring well C1 and C2 use to be overflowed by water during the period in water delivery process, and the groundwater in C2 is also too sensitive to the channel water because it is too near to the channel, many monitored data from C1 and C2 were distortion. In addition, the cover of monitoring well C3 often was locked because of some reason and no data record had been obtained for a long time since 2003. So this paper mainly used the observed data for the depth of groundwater in well C4, C5, C6, C7 and C8.

III. METHODS

Grey system theory uses a black-grey-white color spectrum to describe a system [5]-[7], which regards unknown or uncertain information as black information, known or certain information as white information, and the system whose information partially known or known with uncertainty is a grey system. The grey system theory is a multidisciplinary theory dealing with complex systems for which we lack information, whose characteristics are only partially known or known with uncertainty [8]-[10] [13].

Though the dynamic system of groundwater in the lower reaches of Tarim River is a very complex system controlled and affected by many factors, but from the point of view of grey system theory, the system is typical grey system, in which there are three kinds of information, the white information we already know well, the grey information we only know partly, and the black information we do not know at all. So the grey system theory maybe provides us one of methods to approach the problem. As an attempt, this paper use the grey relational analysis method and grey modelling method to study the dynamic of groundwater level in the lower reaches of Tarim River affected by water delivery from upper reaches.

A. Grey relational analysis

It is impossible to establish models for all uncertain factors with grey system method. Therefore, grey relation analysis is involved in selecting most important factors such as the objects for study. In grey relation analysis, influence factors are represented by time series. Grey relation is the indefinite relationship among the time series. The aim of grey relational analysis is to study the affecting degree of comparative series to reference series. A parameter called grey relation degree is used to represent propinquity of two series. If the relation degree of one series is higher than that of others, it places greater influence on reference series, and will be chosen for modelling [6] [8]-[9].

The principle of grey relation analysis is as following: For reference series $\{X_1^{(0)}(t) \mid t = 1,2,\cdots,n\}$ and influence series $\{X_i^{(0)}(t) \mid i = 2,3,\cdots,m; t = 1,2,\cdots,n)$, the following formula is used to calculate relation parameters of two series:

$$\xi_{1i}(t) = \frac{\min\limits_i \min\limits_t \left|X_1(t) - X_i(t)\right| + k \max\limits_i \max\limits_t \left|X_1(t) - X_i(t)\right|}{\left|X_1(t) - X_i(t)\right| + k \max\limits_i \max\limits_t \left|X_1(t) - X_i(t)\right|} \tag{1}$$

where k is a grey parameter is between 0 and 1, which is often assigned a value as 0.5 for calculation.

With relation parameters, we can calculate the grey relation degree of each influence series ($X_i(t)$) to reference series ($X_1(t)$):

$$\gamma_{1i} = \frac{1}{n}\sum_{t=1}^{n} \xi_{1i}(t) \tag{2}$$

It is need to point out that the data of each series should be normalized before doing the grey relational analysis. There are several methods for normalizing the data, and one of methods for this paper normalized the data of each series as following:

$$x_i(t) = \frac{X_i(t) - \min\limits_t X_i(t)}{\max X_i(t) - \min\limits_t X_i(t)} \qquad i = 1,2,\cdots,m \tag{3}$$

That is to say, $X_i(t)$ should be instead by $x_i(t)$ in formula (1) for computation.

B. GM (1, m) model

The grey relational analysis reveals the propinquity relationship for the reference factor $X_1^{(0)}(t)$ among its related factor $X_2^{(0)}(t)$,$\cdots$, $X_m^{(0)}(t)$, but cannot mathematically describe the dynamic relationship among of them. The GM (1, m) model can mathematically describe the dynamic relationship between the characteristic variable and its related factors.

According to the principle of grey modelling [6] [8]-[10], we define the AGO series of $X_2^{(0)}(t)$,$\cdots$, $X_m^{(0)}(t)$ as $X_2^{(1)}(t)$, $X_3^{(1)}(t)$,$\cdots$, $X_m^{(1)}(t)$ respectively, which are generated as following:

$$\begin{aligned} X_2^{(1)}(t) &= \sum_{i=1}^{t} X_2^{(0)}(i), t = 1,2,\cdots,n \\ X_3^{(1)}(t) &= \sum_{i=1}^{t} X_3^{(0)}(i), t = 1,2,\cdots,n \\ &\vdots \\ X_m^{(1)}(t) &= \sum_{i=1}^{t} X_m^{(0)}(i), t = 1,2,\cdots,n \end{aligned} \tag{4}$$

According to the principle of GM (1,m) model, the dynamic relationship between $X_1^{(1)}$ and $X_2^{(1)}$, $\cdots$, $X_m^{(1)}$ can be described by the following differential equation:

$$\frac{dX_1^{(1)}}{dt} + a_1 X_1^{(1)} = \sum_{i=2}^{m} a_i X_i^{(1)} \tag{5}$$

where $a = [a_1, a_2, a_3]^T$ can be obtained by the least squares estimation as

$$a = (B^T B)^{-1} B^T Y \tag{6}$$

in which

$$B = \begin{pmatrix} -\frac{1}{2}[\sum\limits_{i=1}^{2} X_1^{(0)}(i) + \sum\limits_{i=1}^{1} X_1^{(0)}(i)] & \sum\limits_{i=1}^{2} X_2^{(0)}(i) & \cdots & \sum\limits_{i=1}^{2} X_m^{(0)}(i) \\ -\frac{1}{2}[\sum\limits_{i=1}^{3} X_1^{(0)}(i) + \sum\limits_{i=1}^{2} X_1^{(0)}(i)] & \sum\limits_{i=1}^{3} X_2^{(0)}(i) & \cdots & \sum\limits_{i=1}^{3} X_m^{(0)}(i) \\ \vdots & \vdots & & \vdots \\ -\frac{1}{2}[\sum\limits_{i=1}^{n} X_1^{(0)}(i) + \sum\limits_{i=1}^{n-1} X_1^{(0)}(i)] & \sum\limits_{i=1}^{n} X_2^{(0)}(i) & \cdots & \sum\limits_{i=1}^{n} X_m^{(0)}(i) \end{pmatrix}$$

$$Y = [X_1^{(0)}(2), X_1^{(0)}(3), \cdots, X_1^{(0)}(n)]^T$$

The approximate time response function for the differential equation in formula (5) is as following:

$$X_1^{(1)}(k+1) = [X_1^{(0)}(1) - \frac{1}{a_1}\sum_{j=2}^{3} a_j X_j^{(1)}(k+1)]\exp(-a_1 k) + \frac{1}{a_1}\sum_{j=2}^{3} a_j X_j^{(1)}(k+1) \tag{7}$$

IV. RESULTS AND DISCUSSION

Analyzing the groundwater level dynamic affected by water delivery, we found the disciplinary change in each delivery process as: at first the groundwater level in each monitoring well had been gradually ascending in the wake of the transported water arriving from upper reaches when the water delivery started, then gradually falling after the water transmission stopped, finally leveled off at a stable level.

In order to reveal the affection of related factors of water delivery from the upper reaches of Tarim River to the groundwater level on the monitoring section at Yingsu, we calculated the grey relation degree of each influence series (i.e. $X_i^{(0)}(t)$) to reference series (i.e. $X_1^{(0)}(t)$), which

$X_1^{(0)}(t)$ represents the depth of highest-level groundwater caused by t th water delivery process, $X_2^{(0)}(t)$ represents the discharging volume in t th water delivery process, $X_3^{(0)}(t)$ represents the running days in t th water delivery process, $X_4^{(0)}(t)$ represents the daily discharging volume in t th water delivery process, $X_5^{(0)}(t)$ represents the water volume from the mainstream in t th water delivery process, and $X_6^{(0)}(t)$ represents the discharging volume from Bosten Lake in t th water delivery process. The calculated results are shown in table II.

TABLE II
THE GREY RELATION DEGREE FOR THE RELATED FACTORS OF WATER DELIVERY TO THE GROUNDWATER LEVEL

Well	γ_{12}	γ_{13}	γ_{14}	γ_{15}	γ_{16}
C4	0.743	0.8553	0.7124	0.5693	0.6054
C5	0.7537	0.8387	0.719	0.5723	0.6297
C6	0.7541	0.8329	0.7274	0.5806	0.6505
C7	0.8326	0.9181	0.8501	0.6637	0.7839
C8	0.8396	0.8997	0.8569	0.6733	0.8334

From Table II, we found that the descending order for grey relation degree in monitoring well C4, C5 and C6 is as $\gamma_{13} > \gamma_{12} > \gamma_{14} > \gamma_{16} > \gamma_{15}$, and that in monitoring well C7 and C8 is as $\gamma_{13} > \gamma_{14} > \gamma_{12} > \gamma_{16} > \gamma_{15}$. In other words, the order of importance for the factors affecting groundwater in monitoring well C4, C5 and C6 is running days for water delivery, discharging volume, daily discharging volume, discharging volume from Bosten Lake and water volume from the mainstream respectively, and that in monitoring well C7 and C8 is running days for water delivery, daily discharging volume, discharging volume, discharging volume from Bosten Lake and water volume from the mainstream respectively. Though the orders of grey relational degree are a little bit difference, but it is evident that the three major factors related to water delivery from the upper reaches of Tarim River, which affect the depth of groundwater on the monitoring section at Yingsu, are discharging volume, running days for water delivery and daily discharging volume.

From the point of view of logical relation, It is clear that $X_4^{(0)}$ (daily discharging volume) is highly correlated to $X_2^{(0)}$ (discharging volume) and $X_3^{(0)}$ (running days for water delivery), so we should eliminate $X_4^{(0)}$ for mathematically modelling the dynamic relationship between groundwater level and the affected factors.

In order to further describe the dynamic relationship mathematically among $X_1^{(0)}$, $X_2^{(0)}$ and $X_3^{(0)}$, we established the GM (1, 3) models on the monitoring section at Yingsu. The fitted parameters and GM (1, 3) models for monitoring well C4, C5, C6, C7 and C8 are shown in Table III.

Table III shows that all the modelling parameters a_2 and a_3, i.e. the coefficients of $X_2^{(1)}$ and $X_3^{(1)}$ in all equations are positive, which indicate that discharging volume and running days for water delivery from the upper reaches of

TABLE III
GM (1, 3) MODELS FOR MONITORING WELL C4, C5, C6, C7 AND C8

Well	The Distance apart from the river center (m)	Modelling parameters			Equations
		a_2	a_2	a_3	
C4	250	0.2554	0.3711	0.0076	$\frac{dX_1^{(1)}}{dt} + 0.2554X_1^{(1)} = 0.3711X_2^{(1)} + 0.0076X_3^{(1)}$
C5	350	0.5235	0.6680	0.0133	$\frac{dX_1^{(1)}}{dt} + 0.5235X_1^{(1)} = 0.6680X_2^{(1)} + 0.0133X_3^{(1)}$
C6	450	0.8281	1.1290	0.0235	$\frac{dX_1^{(1)}}{dt} + 0.8281X_1^{(1)} = 1.1290X_2^{(1)} + 0.0235X_3^{(1)}$
C7	750	1.044	1.7317	0.0345	$\frac{dX_1^{(1)}}{dt} + 1.044X_1^{(1)} = 1.7317X_2^{(1)} + 0.0345X_3^{(1)}$
C8	1050	1.7544	2.9757	0.0655	$\frac{dX_1^{(1)}}{dt} + 1.7544X_1^{(1)} = 2.9757X_2^{(1)} + 0.0655X_3^{(1)}$

Tarim River are two major factors that markedly control and raise the groundwater level.

From Table III, we also find all of the modelling parameter a_1, i.e. the coefficients of $X_1^{(1)}$ in all equations are positive and appear gradually increasing trend versus distance apart from river center, which imply that the response degree of $\frac{dX_1^{(1)}}{dt}$ to $X_1^{(1)}$ reduces gradually versus the increase of the distance apart from river center. In other words, the change rate of groundwater level respond to itself becomes more and more insensitive with increase of the distance apart from river center, which is corresponding to the study conclusions of Chen et al [12] and Xu et al [4].

We also know that from Table III the modelling parameter a_2 and a_3, i.e. the coefficients of $X_2^{(1)}$ and $X_3^{(1)}$ in all equations gradually increase versus the distance apart from river center, which show that the change rate of groundwater level ($\frac{dX_1^{(1)}}{dt}$) respond to discharging volume ($X_2^{(1)}$) and running days for water delivery ($X_3^{(1)}$) increase versus the distance apart from river center. In other words, the affection from discharging volume and running days for water delivery to the change rate of groundwater level becomes more and more significant with increase of the distance apart from river center. The results imply that water delivery not only markedly controls and raises the groundwater level near river, but also affects the groundwater level as far as the range in the distance of 1050 m apart from river center.

V. CONCLUSIONS

Through demonstration study on the monitoring section at Yingsu by grey relational analysis and grey modelling, we think that the grey system theory is really one of the methods for approach to the dynamic of groundwater level in the lower reaches of Tarim River affected by transported water from upper reaches.

From above research results, we elicit the following conclusions:

(1) The results of grey relational analysis show that discharging volume, running days for water delivery and daily discharging volume, which related with water delivery from the upper reaches of Tarim River, are three main factors that markedly control and affect the groundwater level.

(2) The Modelling results based on the GM (1, 3) model show that the change rate of groundwater level respond to itself becomes more and more insensitive with increase of the distance apart from river center, and the affection from discharging volume and running days for water delivery to the change rate of groundwater level becomes more and more significant with increase of the distance apart from river center. Water delivery not only markedly controls and raises the groundwater level near river, but also affects the groundwater level as far as the range in the distance of 1050 m apart from river center.

REFERENCES

[1] X. Li, D.Yang. Benefit and ecological loss of water utilization in Tarim River. Arid Land Geography (in Chinese), 2001, 24(4): 327–331.

[2] Y. Chen, W. Cui, W. Li, et al. Exploitation of water resources and ecology protection in Tarim River. Acta Geographica Sinica (in Chinese), 2003, 58(2): 215–222.

[3] Y. Chen, H. Zilliacus, W. Li, et al. Ground-water level affects plant species diversity along the lower reaches of the Tarim River, Western China. Journal of Arid Environments, 2006, 66: 231-246.

[4] H. Xu, Y. Song, Y. Chen. Study on variation of groundwater after ecological water transport in the lower reaches of Tarim River. Advance in water science (in Chinese), 2004, 15(2): 223–226.

[5] J. Deng. Control problems of grey system. System & Control Letters, 1982, 1(5): 288–294.

[6] J. Deng. Grey system: society and economics (in Chinese). Beijing: National defence industry press, 1985, 1–272.

[7] J. Deng. Introduction to grey system. Journal of Grey System, 1989 1(1): 1–24.

[8] S. Liu, J. Forrest. The role and position of Grey system theory in science development. The Journal of Grey System, 1997, 9(4): 351-356.

[9] K. Wen. Grey Systems: Modeling and Prediction. Yang's Scientific Press, Tucson, USA. 2004, 145-218.

[10] R. Guo. Repairable System Modeling Via Grey Differential Equations. Journal of Grey System, 2005, 8(1): 69-91.

[11] Y. Chen, Q. Wang, W. Li, et al. Rational groundwater table indicated by the eco-physiological parameters of the vegetation: A case study of ecological restoration in the lower reaches of the Tarim River. Chinese Science Bulletin, 2006, 51(Supp. I): 8 -15.

[12] Y. Chen, X. Zhang, W. Li, et al. Analysis on the ecological benefits of the stream water conveyance to the dried-up river of the lower reaches of Tarim River, China. Science in China (D), 2004, 47(11): 1053–1064.

[13] S. Liu, Y. Dang, Z. Fang, et al. Grey system theory and application (in Chinese). Beijing: Science Press, 2004, 1-376.

[14] J. Xu, Y. Chen, W. Li. Long-term Trend and Fractal of Annual Runoff Process in Mainstream of Tarim River. Chinese Geographical Science, 2008, 18(1): 77-84.

[15] J. Xu. Mathematical Methods in Contemporary Geography (in Chinese). Beijing: Higher Education Press, 2002, 338-369.