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# Signal Processing for the annual runoff process by wavelet and R/S analysis: A case study of the Tarim headwater basin

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*Abstract*: **The paper studies signal processing for the annual runoff processes of the Tarim headwater basin. By utilizing the wavelet and R/S analysis, the main findings are generalized as follows: (1) The annual runoff processes of the three headwaters of the Tarim River are complex nonlinear systems, which show nonlinear trends, as well as fractals, and long-term correlation. (2) In the time scale of 16 ($2^4$) years from 1957 to 2002, the annual runoff in the Aksu and Yarkand Rivers show an increasing trend in general, while in the Hotan River a slightly decreasing trend is observed for the same time period. If the time scale reduces to 8 ($2^3$) or 4 ($2^2$) years, the annual runoff in each river does not show an apparent trend of either increasing or decreasing. (3) The time series of annual runoff in each river present a long-term correlation characteristic. The Hurst exponent in the period from 1989 to 2002 indicates that the annual runoff in the Aksu and Yarkand River will show an increasing trend, and that in the Hotan River will show a decreasing trend in the years after 2002.**

*Key words:* **annual runoff process; Tarim headwater basin; wavelet; R/S analysis**

## I. INTRODUCTIONS

The largest continental river basin in China, Tarim River Basin is rich in natural resources, but has a vulnerable ecological environment. The contradiction between ecological protection and economic development has become more outstanding in the utilization of water resource. The sustainable development of the regional economy in the basin is severely restricted by water resource [1-2], and a majority of available water resource comes from the runoff of river flows. However, studies [3-5] showed that the runoff processes of many river flows are complex systems, which show nonlinear trends as well as fractals characteristic.

The objective of this study is to signal processing for the annual runoff process by wavelet and R/S analysis, and reveal their nonlinear trends as well as fractal characteristics of annual runoff process of the three headwaters of the Tarim River, i.e. The Aksu River, Yarkand River and Hotan River. Based on the time series from 1957 to 2002.

## II. STUDY AREA AND DATA

### A. Study Area

The Tarim River Basin, with an area of $1.02\times10^6$ km$^2$, is the largest continental river basin in China. It covers the entire southern part of Xinjiang in western China that is characterized by both rich natural resources and fragile environment, and has an extreme desert climate with an average annual temperature of 10.6 ~ 11.5 °C. Monthly mean temperature ranges from 20 to 30 °C in July and -10 to -20 °C in January. The highest and lowest temperatures are +43.6 °C and -27.5 °C respectively. The accumulative temperature >10 °C ranges from 4100 to 4300 °C. Average annual precipitation is about 116.8 mm in the whole area, and the figure ranges from 200 to 500 mm in the mountainous area, 50 to 80 mm on the edges of the basin and only 17.4 ~ 25.0 mm in the central area of the basin. There is great unevenness in the precipitation distribution within any year. More than 80% of the total annual precipitation falls between May and September in the high-flow season, and less than 20% of the total falls from November to April.

The main channel of the Tarim River is 1321 km in length. Naturally and historically the Tarim River Basin consists of 114 rivers from nine drainage systems, which include Aksu, Hotan, Yarkand, Qarqan, Keriya, Dina, Kaxgar, Kaidu–Konqi Rivers. The basin covers an arable land area of 20.44×106 ha and has a human population of 8.26 × 106. Mean annual natural surface runoff is 3.98 × 1010 m3, which originates mostly from glaciers, snowmelt and precipitation in the surrounding mountains.

Intensive disturbances caused by human activities, particularly regarding excessive water resources exploitation, have brought about marked changes during the past 50 years. The drainage systems gradually disintegrated when the Weigan, Kaxgar, Dina, Keriya, and Qarqan Rivers stopped flowing to the mainstream and were eventually disconnected from it. Today, there are only three drainage systems connected to the mainstream of the Tarim River. These are the Aksu River, Yarkand River and Hotan River. Having two main sub-streams, Tongshigan and Kumalak rivers, the Aksu River runs from the Tianshan Mountains in the northwest of the basin. The Hotan River, having two main sub-streams, the Kalaksh and Yulongkash rivers, originates from the Kunlun Mountains and is located in the southwest of the basin. The Yarkand River originates from the Pamir Plateau and lies between the Hotan and Aksu Rivers.

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### B. Data

To analyze the long-term trend for annual runoff in the three headwaters of the Tarim River, the runoff data from 1957 to 2002 in five sub-rivers are used. They are respectively the data of The Aksu River from Xiehela and Shaliguilank hydrologic station, the data of The Yarkand River from Kaqun hydrologic station, and the data of The Hotan River from Tonguzluok and Wuluwat hydrologic station. As all the stations are located in the source areas of these rivers, and water use by humans within every river basin is in a relatively small quantity compared to the total discharge, the monitoring hydrological data reflect approximately the natural condition.

Because Long-term climate change can alter the pattern of runoff production, the timing of hydrological events and the frequency and severity of floods, particularly in arid or semi-arid regions, a small change in precipitation and temperature may result in marked changes in runoff [7]. For such reasons, the data on annual runoff in the three headwaters of the Tarim River from 1957 to 2002 are used in this study.

## III. METHODS

### A. Wavelet analysis

Wavelet analysis is a multi-resolution analytical approach. It is a way to analyze the time scales of signals [8-10]. It may well be able to provide new insights into the periodicity of runoff processes. By using wavelet analysis method, this paper will reveal the nonlinear trend of annual runoff process in the three headwaters of the Tarim River at different time scale.

Considering the time series of annual runoff in a certain river X(t), which can be built up, as a sequence of projections onto Father and Mother wavelets indexed by both $\{k\}$, $k=\{0,1,2\ldots\}$ and by $\{s\}$, $s=2^j$, $j=\{1,2,3\ldots\}$.

The coefficients in the expansion are given by the projections

$$s_{J,k} = \int X(t)\Phi_{J,k}(t)dt \qquad (1)$$

$$d_{j,k} = \int X(t)\Psi_{j,k}(t)dt, j = 1,2,\ldots,J \qquad (2)$$

where $J$ is the maximum scale sustainable by the number of data points, $\Phi_{j,k} = 2^{-j/2}\Phi(\frac{t-2^j k}{2^j})$ is father wavelet, and $\Psi_{j,k} = 2^{-j/2}\Psi(\frac{t-2^j k}{2^j})$ is mother wavelet. Generally, father wavelet is used for the lowest-frequency smooth components, which requires wavelet with the widest support; mother wavelet is used for the highest-frequency detailed components. In other words, father wavelet is used for the major trend components, and mother wavelet is used for all deviations from the trend.

The representation of the signal X(t) now can be given by:

$$X(t) = \sum_k s_{J,k}\Phi_{J,k}(t) + \sum_k d_{J,k}\Psi_{J,k}(t) + \sum_k d_{J-1,k}\Psi_{J-1,k}(t) + \ldots\ldots + \sum_k d_{1,k}\Psi_{1,k}(t) \quad (3)$$

We can represent the approximation in a more revealing manner for our purposes:

$$X(t) = S_J + D_J + D_{J-1} + \ldots D_j + \ldots + D_1 \qquad (4)$$

where $S_J = \sum_k s_{J,k}\Phi_{J,k}(t)$

and $D_j = \sum_k d_{j,k}\Psi_{j,k}(t), j = 1,2,\ldots J.$

In general, we have

$$S_{j-1} = S_j + D_j \qquad (5)$$

where $\{S_J, S_{J-1}, \ldots, S_1\}$ is a sequence of multi-resolution approximations of the function X(t) at ever-increasing levels of refinement. The corresponding multi-resolution decomposition of X(t) is given by $\{S_J, D_J, D_{J-1}, \ldots, D_j, \ldots, D_1\}$.

One of our main interests of this research is to approximate the trends based on decomposition and reconstruction for the time series of annual runoff by using the above wavelet analysis method at different time scales. One of main objectives of this study is to examine the fluctuations for the time series of annual runoff in the three headwaters of the Tarim River by different time scales, and to have an insight into the overall variation of the signal with time. We choose the Symmlet as the basic wavelet, designated ‘Sym8’. We experimented with alternative choices of scaling functions and of wavelet, but the qualitative results were very robust to such changes and the initial choice of wavelet seemed to be the best on balance [17]. In all cases, the levels analyzed are restricted to S4, S3, and S2 to represent the trend elements.

### B. R/S analysis

All statistical methods are based on the same assumption that all data of time series are independent (i.e. fit for Gauss distribution), hence the series is stochastic. When HE Hurst [11-12], a British physicist, analyzed water level of the Nile River, he found that such time series of variables such as river water level were not fit for Gauss distribution, showing a characteristics of discontinuity and durability. Based on the empirical findings of Hurst, BB Mandelbrot [13-14] made a breakthrough regarding fundamental theories of traditional statistical methods. He found many time series no longer present a random Brownian movement unrelated to the past, but show a characteristic of long-term correlation [15], which he called “fractal”.

In order to study the fractal characteristic of annual runoff processes in the three headwaters of the Tarim River, the rescaled range (R/S) analysis method are used in our research work.

Considering the time series of annual runoff in a certain river, X(t), for any positive integer $\tau \geq 1$, the mean value series is defined as

$$\langle X\rangle_{\tau}=\frac{1}{\tau}\sum_{t=1}^{\tau}X(t)\quad \tau=1,2,\cdots \qquad (6)$$

The accumulative deviation is

$$X(t,\tau)=\sum_{u=1}^{t}(X(u)-\langle X\rangle_{\tau})\quad 1\le \mathrm{t}\le \tau \qquad (7)$$

The extreme deviation is

$$R(\tau)=\max_{1\le t\le \tau}X(t,\tau)-\min_{1\le t\le \tau}X(t,\tau)\quad \tau=1,2,\ldots \qquad (8)$$

The standard deviation is

$$S(\tau)=\left[\frac{1}{\tau}\sum_{t=1}^{\tau}(X(t)-\langle X\rangle_{\tau})^{2}\right]^{\frac{1}{2}}\quad \tau=1,2,\cdots \qquad (9)$$

When analyzing the statistic rule of $R(\tau)/S(\tau)\triangleq R/S$ , HE Hurst discovered a relational expression

$$R/S\propto(\frac{\tau}{2})^{H} \qquad (10)$$

which shows there is Hurst phenomenon in the time series, and where $H$ is called the Hurst exponent.

Apparently, the Hurst exponent $H$ is given by the slope coefficient of R/S versus τ/2. According to (τ, R/S), $H$ can be obtained by least squares method (LSM) in a log-log grid.

Hurst *et al* (1965) once proved that if {X(t)} is an independently random series with limited variance, the exponent $H$=0.5; and $H$ ($0<H<1$) is dependent on an incidence function $C(t)$:

$$C(t)=2^{2H-1}-1 \qquad (11)$$

When $H$>0.5, C(t)>0, it means that the process has a long-enduring characteristic, and the future trend of the time series will be consistent with the past. In other words, if the past showed an increasing trend, the future will also show an increasing trend. When $H$<0.5, C(t)<0, it means that the process has an anti-persistence characteristic, and the future trend of the time series will be opposite from the past. In other words, if the past showed an increasing trend, the future will assume the reducing trend. When $H$=0.5, C(t)=0, it means that the process is stochastic. In other words, there is no correlation or only a short-range correlation in the process [16-17].

## IV. RESULTS AND DISCUSSION

### A. Nonlinear trends of annual runoff processes

Based on decomposition and reconstruction for the time series by using the above wavelet analysis method at different time scales, we found that the nonlinear trends of the annual runoff processes in the three headwaters of the Tarim River are different at different time scales.

Fig. 1 shows the wavelet approximation curves of the annual runoff in the Aksu River by different time scales. At the time scale of S4, i.e. the time scale of 16 ($2^4$) years, the curve shows a local as well as the globe maximum point appeared in 1965 with the value of 71.2597×$10^8$m$^3$, and a local as well as the globe minimum point appeared in 1975 with the value of 70.1458×$10^8$m$^3$. Though the curve fluctuated slightly, a basically ascending trend is obvious as a whole from 1957 to 2002. The long-term increasing trend corresponds to the result figured out by Chen et al (2006). If the time scale is reduced to S3, i.e. the time scale of 8 ($2^3$) years, the curve as a whole still presents the basic trend as that at the time scale of S4, but fluctuated obviously during the period. There are two local maximum and minimum points on the curve, which the two local maximums appear respectively in 1966 (72.80×$10^8$m$^3$) and 1979 (75.14×$10^8$m$^3$), and the two local minima appear respectively in 1971 (71.47×$10^8$m$^3$) and 1986 (69.42×$10^8$m$^3$). If the time scale is further reduced to S2, i.e. the time scale of 4 ($2^2$) years, though the curve as a whole still presents the basic trend as that at the time scale of S3, it has fluctuated more obviously during the period. There are five local maximum and four local minimum points on the curve, which the five local maximums appear respectively in 1960 (67.57×$10^8$m$^3$), 1967 (79.62×$10^8$m$^3$), 1979 (78.26×$10^8$m$^3$), 1987 (73.55×$10^8$m$^3$) and 1998 (93.84×$10^8$m$^3$), the four local minima appear respectively in 1962 (66.80×$10^8$m$^3$), 1974 (68.02×$10^8$m$^3$), 1984 (69.71×$10^8$m$^3$) and 1990 (71.57×$10^8$m$^3$).

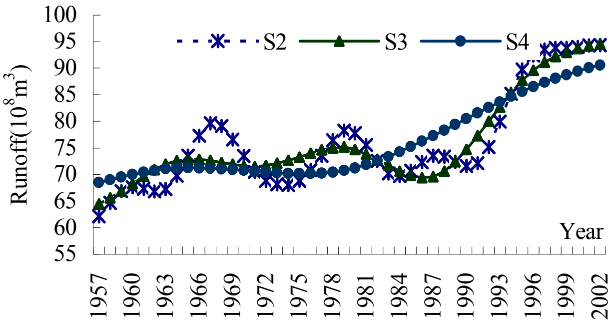

Fig.1 The wavelet approximations for annual runoff in the Aksu River by different time scales

Studying the annual runoff time series in the Yarkand River and Hotan River, we obtained the similar results as Fig. 1. The results are: In the time scale of 16 ($2^4$) years from 1957 to 2002, the annual runoff in the Aksu and Yarkand Rivers show an increasing trend in general, while in the Hotan River a slightly decreasing trend is observed for the same time period. If the time scale reduces to 8 ($2^3$) or 4 ($2^2$) years, the annual runoff in each river does not show an apparent trend of either increasing or decreasing.

### B. The long-term correlation characteristics

By using wavelet analysis, we found that the annual runoff of the mainstream and the three headwaters in the Tarim River basin present increasing or decreasing trend at the time scale of 16 (24) years. If the time extent from 1958 to 2005 was divided into three periods in accordance with the range of 16 (24) years, i.e. 1957~1972, 1973~1988 and 1989~2002, the Hurst exponents for annual runoff of the three headwaters in the Tarim River basin in different periods have been calculated, which are shown in Table 1.

Table 1. Hurst exponents for annual runoff processes in the three headwaters of the Tarim River Bain

| | 1957~1972 | 1973~1988 | 1989~2002 |
|---|---|---|---|
| Aksu River | 0.7632 | 0.5984 | 0.9083 |
| Yarkand River | 0.6425 | 0.4696 | 0.7342 |
| Hotan River | 0.7354 | 0.6415 | 0.5214 |

For the Aksu River, the Hurst exponents in the three periods, 1957 ~ 1972, 1973 ~ 1988 and 1989 ~ 2002 are 0.7632, 0.5984 and 0.9083 respectively. These are all greater than 0.50 and indicate that in every period the annual runoff process showed a long-enduring characteristic. That is to say, the next period will have the same trend with the preceding period. Figure 8 reveals the annual runoff trend that had increased in the first period (1957~ 1972), and slightly increased in the second period (1973 ~ 1988), then increased in the third period (1989 ~ 2002). These trends correspond to the results indicated by the Hurst exponents. According to the Hurst exponent of the period from 1989 to 2002, we can affirm that the annual runoff of The Aksu River can be expected to increase in the 14 years after 2002.

For The Yarkand River, in the period from1957 to1972, the Hurst exponent equals 0.6425, which is greater than 0.50 and means that in the period the annual runoff process displayed an enduring characteristic; in the period from1973 to1988, the Hurst exponent equals 0.4696, which is less than 0.50 and means that the annual runoff process showed anti-persistence characteristic; in the period from1989 to2002, the Hurst exponent equals 0.7342, which is greater than 0.50 and means that in the period the annual runoff process presented an enduring characteristic again. Figure 9 clearly shows the annual runoff had decreased slightly in the period of 1957~1972 and 1973~1988, had increased in the period of 1989~2002. The trends are consistent with the results indicated by Hurst exponents. From the Hurst exponent of the period from 1989 to 2002, we can affirm that the annual runoff of Yarkand will increase slightly in the next 14 years after 2002.

For The Hotan River, the Hurst exponents in the three periods, 1957~1972, 1973~1988 and 1989~2002 are 0.7354, 0.6415 and 0.5214 respectively, which are greater than 0.50 and indicate that in every period the annual runoff process showed a long-enduring characteristic. Figure 10 clearly shows the slightly ascending trend of annual runoff in all the three periods of 1957~1972, 1973~1988 and 1989~2002, which correspond with the Hurst exponents. From the Hurst exponent of the period from 1989 to 2002, we can affirm that the annual runoff of The Hotan River is likely to show a slightly decreasing in the next 14 years after 2002.

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