Using GM (1,1) models to predict groundwater level in the lower reaches of Tarim River: A demonstration at Yingsu section

Jianhua Xu[1] Yaning Chen[2] Weihong Li[2]

1. The Key Lab of GIScience of the Education Ministry PRC, Dept. of Geography, East China Normal University, 200062 Shanghai, People's Republic of China.

E-mail: xujianhua_cn@yahoo.cn

2. Xinjiang Institute of Ecology and Geography, The Chinese Academy of Sciences, Urumqi, Xinjiang 830011, People's Republic of China.

E-mail: chenyn@ms.xjb.ac.cn, liwh@ms.xjb.ac.cn

Abstract

Grey System theory is a multidisciplinary theory dealing with those systems for which we lack information, which uses a black-grey-white color spectrum to describe a complex system whose characteristics are only partially known or known with uncertainty. From the point of view of grey system theory, the dynamic of groundwater level in the lower reaches of Tarim River is typical grey system, which maybe provides us one of methods to approach the problem. As an attempt, through demonstration at Yingsu section, the paper has comparatively studied the grey forecasting models to predict the groundwater level in the lower reaches of Tarim River. The conclusions are: (1) The grey forecasting models, which include equal and unequal time lag GM (1,1) model, are applicable models to predict the groundwater level in lower reaches of Tarim River. The accuracy test parameters, P and C for equal and unequal time lag GM (1,1) model both achieved the desired impact for prediction. (2) The forecasted depth of groundwater for each well by unequal time lag GM (1,1) model is somewhat less than that by equal time lag GM (1,1) model. If the average of forecasted depth of groundwater by the two kind models is regarded as the predicted result, the depth of groundwater of monitoring well C4, C5 and C6 in 2007 will be 4.1696 m, 4.317 m and 4.4839 m respectively, and that in 2008 will be 4.0612 m, 4.2308 m and 4.3604 m respectively.

I. INTRODUCTION

Groundwater is a complex dynamic system. Generally, groundwater levels are dynamic responses of a groundwater system to its input (i.e., recharge) and output (i.e., base flow or groundwater discharge to rivers and streams). The recharge process is affected by various hydrologic processes (i.e., precipitation, evaporation, transpiration, runoff, infiltration, and soil moisture that in turn depend on atmospheric temperature and pressure, solar radiation, wind speed, topography, land use and land cover, etc.) and by the hydraulic properties of soils and aquifers (i.e., hydraulic conductivity and porosity). These natural processes and properties vary at different spatial and temporal scales and their variations affect fluctuations of groundwater levels and base flow directly or indirectly. As a result, groundwater levels and base flow may vary over multiple spatial and temporal scales with no single characteristic spatial and temporal scale [1]. From this it can be seen that modeling the dynamic of groundwater level is a complex problem that requires detailed understanding of the hydrological processes involved [2]. The dynamic of groundwater level in the lower reaches of Tarim River is even more complicated, which is controlled and affected by not only various nature process but also various human activities, especially affected by transported water from upper reaches [3-5].

The 1321-kilometer Tarim River runs west to east along the northern edge of the Taklimakan Desert, China's largest, and flows into Taitema Lake. The river is the most important source of water in semi-arid Xinjiang, with more than 8 million people living in oases clustered along its banks and in an alluvial plain downstream. Yet, with the increasing population, due to excessive water use for irrigation, industrial and living consumption in the upper and middle reaches of Tarim River, its downstream, which extends further down from the Daxihaizi Reservoir, has become completely dry ever since 1972. As a consequence, the ground-water depth in this area has lowered sharply and the ecosystem is damaged seriously. Subsequently, many plant communities gradually diminish or completely die away in a turn of grass, shrub and arbor; the situation of desertification of this region becomes more and more aggravated [6]. With a full length of 428 km, the lower reaches of Tarim River are located in the eastern part of Tarim Basin in Xinjiang. Surrounded by Taklimakan desert and Kuluk desert, as one of the most drought regions in the western part of China, the lower reaches of Tarim River have serious ecological and environmental problems.

In order to prevent desertification, protect and rehabilitate the ecosystem in the lower reaches of Tarim

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River, China central government launched an emergency water diversion program in 2000. So far, there have been 8 times human water diversion project from Daxihaizi Reservoir to lower reaches of Tarim River since May 2000. On 2 November 2001, third water diversion made water flowing into Taitema Lake where is in lower reaches of Tarim River, which was the first time for water moistening the dried land on the bottom of the lake since it had dried for 30 years. Results show that groundwater level in lower reaches of Tarim River has went up obviously under the effect of water diversion [4], but modelling studies on dynamic of the groundwater level are insufficient.

In fact, from the point of view of grey system theory, the dynamic of groundwater level in the lower reaches of Tarim River is typical grey system, which maybe provides us one of methods to attempt the problem [11]. As an attempt in this paper, we use the GM (1,1) to predict the groundwater level in the lower reaches of Tarim River.

II. METHOD

Grey System theory is a multidisciplinary theory dealing with those systems for which we lack information, which uses a black-grey-white color spectrum to describe a complex system whose characteristics are only partially known or known with uncertainty [6] [8].

The dynamic of groundwater level in the lower reaches of Tarim River is controlled and related by many factors, which is a very complicated and lots of them have not known well by people. Form the point of view of grey system theory, the dynamic of groundwater in the lower reaches of Tarim River is typical grey system, and there are three kinds of information, in which the white information we already know well, the grey information we now know partly, and the black information we do not know at all about it. So the grey system theory maybe provides us one of methods to study the system. This paper select one of the monitoring section in lower reaches of Tarim River, the Yingsu section as a representative, attempt to predict the groundwater level by grey prediction method, and compare predicted effects of the equal and unequal time lag GM (1,1) model.

The GM (1,1) model means a single differential equation model with a single variation. The modeling process is as follows: First of all, observed data are converted into new data series by a preliminary transformation called AGO (accumulated generating operation). Then a GM model based on the generated sequence is built, and then the prediction values are obtained by returning an AGO' s level to the original level using IAGO (inverse accumulated generating operation) [7] [9] [10].

Suppose there is a series of discrete nonnegative data as

$$\begin{matrix} x^{(0)}(1) & x^{(0)}(2) & x^{(0)}(3) & \cdots & x^{(0)}(n) \\ t_1 & t_2 & t_3 & \cdots & t_n \end{matrix}$$

where t_i is the time corresponding to the discrete data. When $\Delta t_i = t_i - t_{i-1} \neq const$, the series is called unequal time lag series, and its modeling method of GM (1,1) model is as following [9-10]:

Accumulate the discrete data above once to get a new serial, that is

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(t_i), \quad k = 1,2,\cdots,n \tag{1}$$

Thus, the dynamic process of $x^{(1)}(k)$ can be described as the following differential equation [7] [9] [10]:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = u \tag{2}$$

where $b = [a,u]^T$ can be calculated by the least squares estimation as

$$b = (B^T B)^{-1} B^T Y \tag{3}$$

in which,

$$B = \begin{pmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)]\Delta t_2 & \Delta t_2 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)]\Delta t_3 & \Delta t_3 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)]\Delta t_n & \Delta t_n \end{pmatrix} \tag{4}$$

$$Y = [x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n)]^T \tag{5}$$

Then, the approximate time response function for $x^{(1)}(t)$ is as following:

$$x^{(1)}(t_i) = [x^{(0)}(1) - \frac{u}{a}]\exp(-a(t_i - t_1)) + \frac{u}{a} \tag{6}$$

The predicted value of $x^{(1)}$, $\hat{x}^{(1)}(t_i)$ can be calculated by formula (6), and the predicted value of $x^{(0)}$, $\hat{x}^{(0)}(t_i)$ can be restored as

$$\hat{x}^{(0)}(t_i) = \hat{x}^{(1)}(t_i) - \hat{x}^{(1)}(t_{i-1}) \tag{7}$$

The predicted errors for $x^{(0)}$ are

$$\begin{aligned} \varepsilon^{(0)}(t_i) &= x^{(0)}(t_i) - \hat{x}^{(0)}(t_i) \\ q(t_i) &= \frac{\varepsilon^{(0)}(t_i)}{x^{(0)}(t_i)} \quad i = 1,2,\cdots,n \end{aligned} \tag{8}$$

The mean of $x^{(0)}$ and $\varepsilon^{(0)}$ respectively are:

$$\bar{x}^{(0)} = \frac{1}{n}\sum_{i=1}^{n} x^{(0)}(t_i)$$

$$\bar{\varepsilon}^{(0)} = \frac{1}{n}\sum_{i=2}^{n} \varepsilon^{(0)}(t_i)$$

The variance of $x^{(0)}$ and $\varepsilon^{(0)}$ respectively are:

$$S_1^2 = \frac{1}{n}\sum_{i=1}^{n}[x^{(0)}(t_i) - \bar{x}^{(0)}]^2$$

$$S_2^2 = \frac{1}{n}\sum_{i=2}^{n}[\varepsilon^{(0)}(t_i) - \bar{\varepsilon}^{(0)}]^2$$

The accuracy for prediction can be examined by the error probability

$$P\{|\varepsilon^{(0)}(t_i) - \bar{\varepsilon}^{(0)}| < 0.6745 S_1\} \quad (9)$$

and the ratio of S_2 to S_1, i.e.

$$C = S_2 / S_1 \quad (10)$$

The value ranges of P and C divide the degree of accuracy for GM (1,1) model, the details [7] are shown in Table 1.

Table 1. The Accuracy degree for the GM (1,1) model

Accuracy degree	P	C
Good	>0.95	<0.35
Qualified	>0.80	<0.5
Just qualified	>0.70	<0.65
Disqualification	≤0.70	≥0.65

For the series $\{x^{(0)}(t_i) \mid i = 1,2,\cdots,n\}$, if $\Delta t_i = t_i - t_{i-1} = \Delta = const$, the series is called equal time lag series, and the modeling method of GM (1,1) model above is applicable too. Especially, if $\Delta t_i = t_i - t_{i-1} = 1$, formula (4) will become

$$B = \begin{pmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{pmatrix} \quad (11)$$

III. STUDY AREA AND DATA

A. *Study area*

The overall length of the lower reaches of Tarim River from Qiala to Taitema Lake is 428 km; the channel bed stretches from north to south on alluvial fans, which is located between Taklamakan Desert and Kuluke Desert. The ground surface is relatively flat, the elevation decreasing from north to south. Water seeps from streams into the alluvial fans, which can recharge shallow aquifers [3].

In the past 50 years, because of the excessive exploitation and unreasonable utilization of water resources in the upper reaches of Tarim River basin, the channel flow in a 320-km section in the lower reaches has been cut off since 1970's, which caused drying-up of streams and lakes at the tail of the river. As a consecutive result, the river course broke and groundwater level dropped down greatly, a large area of forest relying on groundwater for their survival and growth have died, which results in the land desertification aggravating and the ecosystem was damaged seriously. In order to prevent desertification, protect and rehabilitate the ecosystem in the lower reaches of Tarim River, China central government launched an emergency water diversion project in May 2000, transporting water along river channel from up reaches lower reaches.

In order to survey the dynamic of groundwater level in the lower reaches of Tarim River, totally 9 monitoring sections along Tarim River were set up, which are Akdun (section A), Yahepu (section B), Yingsu (section C), Abudali (section D), Kardayi (section E), Tugmailai (section F), Alagan (section G), Yganbjima (section H) and Kaogan (section I). There are several monitoring wells at each section, and there have been 40-65 times aperiodical observation for groundwater level data in each well since May 2000.

Maybe To figure down the dynamic of groundwater level in all lower reaches of Tarim River based on limited observed data is difficult. This paper only chooses the section C, i.e. Yingsu as a focal section, and attempts to predict groundwater level with comparing the forecast results of equal and unequal time lag models.

The section C is 45 km away from Daxihaizi Reservoir along Tarim River, and the running water for every times water delivery crossed here. We used the observed data of groundwater level in 3 monitoring wells, i.e. C4, C5 and C6 for 7 years since 2000. The elevation, latitude, longitude, and the distance apart from river center of each monitoring well are showed in Table 2.

Table 2. Elevation, latitude, longitude, and distance apart from river center of monitoring wells

Monitoring Well	C4	C5	C6
Elevation	845	852	850
Latitude	N40° 25.869'	N40° 25.817'	N40° 25.766'
Longitude	E87° 56.437'	E87° 56.418'	E87° 56.398'
Distance apart from river center (m)	250	350	450

B. *Data inspection*

Table 3 reveals the observed data for depth of groundwater in each monitoring well at Yingsu section. Table 3 tells us the observed data sequence for depth of groundwater in each monitoring well is an unequal time lag series.

Table 3. The observed data for depth of groundwater in each monitoring well at Yingsu section

Observed date	The order day t_i	Time lag ΔT	The depth of groundwater (m)		
			C4	C5	C6
2000-12-31	365	365	5.97	7.02	7.66

2001-11-16	685	320	4.57	4.58	5.42
2002-11-10	1044	359	4.94	4.91	5
2003-11-5	1404	360	4.17	4.38	4.48
2004-12-11	1805	401	5.04	4.94	5.02
2005-11-20	2139	334	4.25	4.44	4.76
2006-12-14	2528	389	4.36	4.65	4.92

From the point of view of the grey system theory, grey prediction is based on the inertia of series. For a series, its inertia is reflected in the data sequence changing, and slight data sequence changing indicates strong inertia. According to the grey system theory, if the stepwise ratio

$$\sigma^{(0)}(k)=\frac{x^{(0)}(k+1)}{x^{(0)}(k)} \qquad k=1,2,\cdots,n \tag{12}$$

satisfied the condition $\sigma^{(0)}(k)\in(0.1353,7.389)$, the inertia of the series achieved the requirement for setting up GM (1,1) model. The more near to 1 $\sigma^{(0)}$ is, the stronger the inertia of the series is. $\sigma^{(0)}(k)=1$ for all k means the inertia of the series is infinity.

Table 4 tells us all the stepwise ratios of each series satisfy the condition to set up GM (1,1) model.

Table 4. The stepwise ratios of each series

C4	C5	C6
$\sigma^{(0)}$	$\sigma^{(0)}$	$\sigma^{(0)}$
0.7655	0.6524	0.7076
1.0810	1.0721	0.9225
0.8441	0.8921	0.8960
1.2086	1.1279	1.1205
0.8433	0.8988	0.9482
1.0259	1.0473	1.0336

Another condition for setting up unequal time lag GM (1,1) model is that the ratio of maximum time lag to minimum time lag is less than 2. Using the data in the third column of Table 3, through simple calculation, we easily know the condition is satisfied too.

The upper data inspection for the conditions to set up GM (1,1) model confirms that all the data sequences are applicable for modeling of grey prediction.

IV. RESULTS AND DISCUSSION

A. *Unequal time lag GM (1,1) models*

Using the method above, we set up unequal time lag GM (1,1) models to describe the dynamic of the depth of groundwater. Table 5 reveals the modeling parameters $b=[a,u]$ and accuracy test parameters (P, C) of unequal time lag GM (1,1) model for each well.

Table 5. Unequal time lag GM (1,1) model to describe the depth of groundwater for each well

Well	Modelling parameters		Accuracy test parameters	
	a	u	P	C
C4	0.0001095	0.01483	0.8571	0.4117
C5	0.000078	0.01454	1.0000	0.2452
C6	0.0001124	0.01621	1.0000	0.4219

B. *Equal time lag GM (1,1) models*

If the time measuring unit is used as "year" instead of "day", the time lag in each sequences may be approximatively regarded as 1, so all the series become equal time lag series. Thus using the method above, we set up equal time lag GM (1,1) models to describe the dynamic of the depth of groundwater for each well, Yingsu. Table 6 reveals the modeling parameters $b=[a,u]$ and accuracy test parameters (P, C) of equal time lag GM (1,1) model for each well at section C, Yingsu.

Table 6. Equal time lag GM (1,1) model to describe the depth of groundwater for each well

Well	Modelling parameters		Accuracy test parameters	
	a	u	P	C
C4	0.0139495	4.8315	0.7143	0.5386
C5	0.003495	4.7222	1.0000	0.2630
C6	0.016002	5.2963	1.0000	0.2538

C. *Comparing the results between equal and unequal time lag GM (1,1) models*

Comparing Table 5 and Table 6 to Table 1, we found that the accuracy test parameters, P and C for equal and unequal time lag GM (1,1) model both achieved the desired impact for prediction. That shows the grey forecasting models, which include equal and unequal time lag GM (1,1) model, are applicable models to predict the groundwater level in lower reaches of Tarim River.

From the accuracy degree of the forecast models, the difference is as follows: For well C5 and C6, the values of accuracy test parameters, P and C achieved the "good" degree for both equal and unequal time lag GM (1,1) models. But for well C4, accuracy test parameters, P and C for the unequal time lag GM (1,1) model achieved the "qualified" degree, and that for equal time lag GM (1,1) model achieved the "just qualified" degree.

Using predictor formula in Table 5 and Table 6, we can calculate the forecast value of the depth of groundwater for each well in 2007~2008, the results are in Table 7.

Table 7 tells us the forecasted depth of groundwater for each well by unequal time lag GM (1,1) model is somewhat less than that by equal time lag GM (1,1) model. It follows that the predicted results for the depth of groundwater should be grey data (Deng, 1982,1985), i.e. interval data. It is evident that the depth of groundwater of monitoring well C4, C5 and C6 in 2007 will drop into bounded interval [4.0025, 4.3367], [4.2325, 4.4015] and [4.3055, 4.6622], and that in 2008 will drop into bounded interval [3.8457, 4.2766], [4.1137, 4.3478] and [4.1325, 4.5882]. Maybe the average of forecasted depth of groundwater by the two kind models will be more close to the real of future groundwater level. If the average of forecasted depth of groundwater by the two kind models is regarded as the predicted result, the depth of groundwater of monitoring well C4, C5 and C6 in 2007 will be 4.1696 m, 4.317 m and 4.4839 m respectively, and that in 2008 will be 4.0612 m, 4.2308 m and 4.3604 m respectively.

Table 7. The forecasted depth of groundwater by equal and unequal time lag GM (1,1) model

Well	Model kind	Forecasted depth of groundwater by the two kind models (m)		The average of forecasted depth of groundwater by the two kind models (m)	
		2007	2008	2007	2008
C4	Unequal time lag	4.0025	3.8457	4.1696	4.0612
	Equal time lag	4.3367	4.2766		
C5	Unequal time lag	4.2325	4.1137	4.317	4.2308
	Equal time lag	4.4015	4.3478		
C6	Unequal time lag	4.3055	4.1325	4.4839	4.3604
	Equal time lag	4.6622	4.5882		

V. CONCLUSIONS

Grey System theory is a multidisciplinary theory dealing with those systems for which we lack information, which uses a black-grey-white color spectrum to describe a complex system whose characteristics are only partially known or known with uncertainty. From the point of view of grey system theory, the dynamic of groundwater level in the lower reaches of Tarim River is typical grey system, which maybe provides us one of methods to approach the problem. As an attempt, through demonstration at Yingsu section, the paper has comparatively studied the grey forecasting models to predict the groundwater level in the lower reaches of Tarim River. Summarizing the study results, we elicit the following conclusions:

(1) The grey forecasting models, which include equal and unequal time lag GM (1,1) model, are applicable models to predict the groundwater level in lower reaches of Tarim River. The accuracy test parameters, P and C for equal and unequal time lag GM (1,1) model both achieved the desired impact for prediction.

(2) The forecasted depth of groundwater for each well by unequal time lag GM (1,1) model is somewhat less than that by equal time lag GM (1,1) model. If the average of forecasted depth of groundwater by the two kind models is regarded as the predicted result, the depth of groundwater of monitoring well C4, C5 and C6 in 2007 will be 4.1696 m, 4.317 m and 4.4839 m respectively, and that in 2008 will be 4.0612 m, 4.2308 m and 4.3604 m respectively.

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