Abstract—The prediction of urban water demand using a small number of representative properties is fundamental in evaluating carrying capacity of water resources. Artificial neural networks (ANNs) have recently become popular tools in the prediction of urban water demand. In this paper, an iterative method which combining the strength of back-propagation (BP) in weight learning and genetic algorithms’ capability of searching the satisfying solution is proposed for optimizing wavelet neural networks (WNNs). Taking the city of Hefei in China as an example, the proposed genetic algorithms optimized WNN that required a few representative properties as possible for input data is applied to predict urban water demand in the future several years. The prediction performance of the GA Optimized WNN is compared with traditional neural networks, and simulation results demonstrate the accuracy and the reliability of the prediction methodology based on the proposed model. Finally, urban water demand in Hefei, 2008-2010, is obtained which provide reference for coordinated development of socio-economic and water resources in Hefei.

Keywords-wavelet neural network; genetic algorithms; prediction; carrying capacity; water resources; Hefei

I. INTRODUCTION

Water is indispensable and fundamental resources for national or regional socio-economic development and ecological environment construction, and it is also regional strategic economic resources for sustainable development. With population growth and economic development, water scarcity is increasingly becoming a crisis that constraints regional sustainable development. Study on carrying capacity of water resources is foundation content of regional sustainable development. Prediction using a small number of representative properties is a practically useful tool for urban water management planning. In the previous works, different methods based on classic methods and intelligent methods for predicting water demand are introduced in [4-5]. But regarding to nonlinear structure of, stochastic parameters of water system and other uncertainties in system, the intelligent methods are more attractive than classic methods.

The ability of nonlinear approximation for model-free estimation of artificial neural networks (ANNs) has been shown by many researchers, as in [6]. Wavelet transforms have emerged as a means of representing a function in a manner which readily reveals properties of the function in localized regions of the joint time-frequency space. The concept of wavelet neural networks (WNNs) was inspired by both the technologies of wavelet decomposition and neural networks. The combination of wavelets with neural networks can hopefully remedy each others weaknesses. The optimal wavelet network structure will achieve the best approximation and prediction capability, and the local optima problem of gradient-based algorithm is avoided, as in [7-10].

A genetic algorithm (GA) is a directed random search technique that is widely applied in optimization problems. This is especially useful for complex optimization problems where the number of parameters is large and the analytical solutions are difficult to obtain. An iterative method which combining the strength of back-propagation (BP) in weight learning and Gas’ capability of searching the satisfying solution is proposed for optimizing WNN. The use of Gas to aid in ANN learning has been a popular approach to address the local optima and design problem of ANN in [11]. In this paper, the proposed GA optimized Wavelet Neural Network
is applied to evaluate carrying capacity of water resources. Taking the city of Hefei as an example, urban water demand prediction model is introduced to reflect the water resources carrying capacity based on the relationship between urban water supply and demand balance. Research results provide reference to coordinated development of regional socioeconomic and water resources in Hefei.

II. GA OPTIMIZED WNN MODEL

A. Wavelet Analysis

Wavelets are obtained from a single prototype wavelet called mother wavelet by dilations and translations:

\[ \phi_{a,b}(x) = \frac{1}{\sqrt{|a|}} \phi\left( \frac{x-b}{a} \right) \]  

(1)

Where \( \phi_{a,b}(x) \) is a family of wavelets, \( a \) is the dilation parameter and \( b \) is the translation parameter, they are real numbers in \( R \) and \( R^+ \) respectively.

The most important properties of wavelets are the admissibility and the regularity conditions and these are the properties, which gave wavelets their name. It is square integrable function \( \phi(t) \) satisfying the admissibility condition:

\[ \int_{-\infty}^{+\infty} \hat{\phi}(\sigma)^2 \left| \phi(\sigma) \right|^2 d\sigma < +\infty \]  

(2)

\( \hat{\phi}(\sigma) \) stands for the Fourier transform of \( \phi(t) \). Any function of \( L^2(R) \) can be approximated to any prescribed accuracy with a finite sum of wavelets. Therefore, wavelet networks can be considered as an alternative to neural and radial basis function networks.

B. Genetic Algorithms

Genetic algorithms (GAs) are adaptive heuristic search algorithms based on the evolutionary ideas of natural selection and genetic. They are modeled loosely on the principles of the evolution via natural selection, employing a population of individuals that undergo selection in the presence of variation-inducing operators such as inheritance, mutation, selection, and crossover(also called recombination). A fitness function is used to evaluate individuals, and reproductive success varies with fitness. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem.

The algorithm can be summarized as follows:

Step 1: Randomly generate an initial population \( M(0) \);

Step 2: Compute and save the fitness \( u(m) \) for each individual \( m \) in the current population \( M(t) \);

Step 3: Define selection probabilities \( p(m) \) for each individual \( m \) in \( M(t) \) so that \( p(m) \) is proportional to \( u(m) \);

Step 4: Generate \( M(t+1) \) by probabilistically selecting individuals from \( M(t) \) to produce offspring via genetic operators;

Step 5: Repeat step 2 until satisfying solution is obtained.

C. Wavelet Neural Network Model

The concept of wavelet networks was introduced by Zhang and Benveniste in [7] as a computational scheme that combines the mathematical rigor of wavelet theory with the adaptive learning properties of conventional neural networks into a single unit. Recently, different types of wavelet neural networks (WNNs) have been proposed. The structure of the wavelet based neural network is similar to that of BP NN, except that here the activation function of the hidden nodes is replaced by wavelet functions. For approximation and forecasting the wavelet network should have a better performance than the traditional neural network. The structure of a wavelet network is shown in Fig. 1.

Employing the wavelets function \( \phi_{ab}(x) \) as the activation functions of the hidden layer nodes, the output of the kth unit in the output layer of the proposed network is given as:

\[ \hat{y}_k(t) = f \left( \sum_{j=1}^{m} \nu_{jk} \cdot \phi_{ab} \left[ \sum_{i=1}^{n} w_{ij} \cdot x_i(t) + \theta_j \right] + r_k \right) \]  

(3)

\( t = 1, 2, \ldots, p \)

Where \( x_i \) is the ith input vector, \( \hat{y}_k \) is the kth output vector, \( \omega_{ij} \) is the synaptic weight connecting the input layer node \( i \) and hidden layer node \( j \), \( \nu_{jk} \) is the synaptic weight connecting the output layer node \( k \) and hidden layer node \( j \). \( \theta_j \) is the bias of the hidden layer node \( j \), \( r_k \) is the bias of the output layer node \( k \). \( m (i = 1, 2, \ldots, m) \) is the number of neurons in the input layer, \( s (j = 1, 2, \ldots, s) \) is the number of neurons in the hidden layer, \( n (k = 1, 2, \ldots, n) \) is the number of neurons in the output layer.

Sigmoid function \( f \) is:

\[ f(x) = \frac{1}{1 + e^{-x}} \]  

(4)

Set \( net_j = \sum_{i=1}^{m} w_{ij} \cdot x_i(t) + \theta_j \),

(5)

For each hidden node a function of the form

\[ \phi_{ab}(net_j) = \phi\left( \frac{net_j - b}{a_j} \right) \]  

(6)
is assigned. where $a_j$ is the dilation parameter of the $j$th hidden node, $b_j$ is its translation parameter.

The basic mother wavelet function used in this work is the real-valued Morelet wavelet.

$$\varphi(x) = \cos(1.75x) \exp(-\frac{x^2}{2})$$ (7)

WNN have four main parameters: dilation ($a_j$), translation ($b_j$), bias ($\theta_j, r_k$), and weights ($\omega_j, V_{jk}$). The values of these parameters are learned concurrently during the neural network training process.

An error vector can be defined as the difference between the network output $\hat{y}_k$ and the desired output $y_k$. As usual, the training is based on the minimization of the following cost function:

$$E(t) = \frac{1}{2} \sum_{n=1}^{N} \sum_{j=1}^{L} [y_k(t) - \hat{y}_k(t)]^2 \leq \epsilon$$ (8)

Where, $\epsilon$ is a given error in advance. Gradient methods have been applied for adjustable parameters. As any gradient-descent based algorithm, the error back propagation algorithm suffers from slow convergence and high probability of converging to local minima. There are effective ways to improve the algorithm by using adaptive learning rates and momentum terms.

The algorithm is stopped when one of several conditions is satisfied: the variation of the gradient, or of the variation of the parameters, reaches a lower bound, or the number of iterations reaches a fixed maximum, whichever is satisfied first.

D. Learning Algorithms

Selecting initial values of the wavelet network parameters are important because initial value affects the speed of the training and approximation to the global or local minimum. So selecting initial values of dilation ($a_j$), translation ($b_j$), bias ($\theta_j, r_k$), and weights ($\omega_j, V_{jk}$) randomly may not be suitable for process modeling.

In this paper, these parameters are initialized to interval 0 and 1, and GAs are used for optimal selection of them before the training procedure. Finally, parameters of the wavelet networks are trained during learning phase for approximation using gradient-descent method.

System modeling consists of four main steps: (1) Input variables and wavelet network structure are determined; (2) Initial values of parameters are randomly selected and these parameters are encoded into the chromosome in GAs; (3) As the learning parameters in WNN, $a_j, b_j, \omega_j, V_{jk}, \theta_j, r_k$ are optimized by using GAs, and the satisfying solution is obtained; (4) Parameters are trained using gradient-descent method during learning phase and simulation results are presented.

Proposed methods are implemented in MATLAB 7 using MATLAB genetic algorithm optimization toolbox (GAOT), the details are illustrated in [12]. Weights, dilation, translation, and bias are encoded into the chromosome in sequence as a solution. The form of code is the following one: $\omega_1, \omega_m, \omega_n, V_1, V_2, \ldots, \theta_1, \theta_m, a_1, a_2, b_1, b_2, \ldots, r_n$.

The fitness function to evaluate a chromosome in the population is written as

$$f = \frac{1}{E}$$ (9)

III. SIMULATION RESULTS

According to the actual situation in Hefei, 14 representative factors affecting the water consumption are selected as follows: agricultural population ($x_1$, ten thousand), non-agricultural population ($x_2$, ten thousand), per capita GDP ($x_3$, yuan), primary industrial outputs ($x_4$, billion yuan), secondary industrial outputs ($x_5$, billion yuan), tertiary industrial outputs ($x_6$, billion yuan), fixed investment ($x_7$, billion yuan), urban per capita annual disposable income ($x_8$, yuan), rural per capita annual net income ($x_9$, yuan), common cultivated area ($x_{10}$, hectare), effective irrigation farmland ($x_{11}$, hectare), irrigation quota of farmland ($x_{12}$, m$^3$ per hectare), water demand of ten thousand industrial added value ($x_{13}$, m$^3$ per thousand), per capita water demand of urban living ($x_{14}$, m$^3$ per capita per day).

Output data are urban water consumption and Data series are currently available for 1970, 1980, 1991, 2001, and 1997-2007. The input and output sequence data are transformed into the domain [0, 1] by normalization for training. The following results are obtained using a WNN with 25 neurons in the hidden layer, the maximum learning iteration is 5000 and the maximum MSE is 0.0001. Momentum coefficient is selected 0.935. The population size used for the GAs is 50, and the maximum generation is 100. The crossover probability is 0.95, and the mutation probability is 0.09.

In order to compare the prediction results of the WNN with traditional neural networks, the training set and testing sets are identical to that in BP NN. Fig. 2 and 3 show the comparative results for GA optimized WNN and BP NN training with TRAINGDA. Fig. 3a provides the network parameter updates using GAs. This demonstrates that after the optimized process of the WNN, the network parameters become stable around the equilibrium state.

It can be seen that the WNN constructed using the proposed approach has a smaller number of iterations and better approximation capabilities than the BP NN. Proposed WNN using GAs achieves a better performance. Simulation results demonstrate the accuracy and the reliability of the prediction methodology based on the proposed model.

| TABLE I. PREDICTIONS OF URBAN WATER DEMAND IN HEFEI |
| --- | --- | --- |
| year | 2008 | 2009 | 2010 |
| Water demand (billion m$^3$) | 1.923 | 2.031 | 2.139 |
Referring to the 11th five-year planning for social-economic development, Urban Master Planning (2006-2020), and the achievement of water planning in Hefei, the input data (2008-2010) are predicted using some statistical methods. Then, we obtain urban water demand in Hefei, 2008-2010, as Table 1 listed.

It can be seen that urban water demand in Hefei shows an increasing tendency between 2008 and 2010, indicating the contradiction between water supply and demand is more serious in the near future. Therefore, we should develop further water saving technology and save water regarding limited water resources in future.

**IV. CONCLUSION**

WNNs are an alternative to neural networks for nonlinear function learning. The GA optimized WNN has achieved the best approximation and prediction capability. Taking the city of Hefei as an example, the proposed genetic algorithms optimized WNN is applied to predict urban water demand. Simulation results demonstrate the accuracy and the reliability of the prediction methodology based on the proposed model. Then, we obtain predictions of urban water demand in Hefei from 2008-2010 based on the network. It can be seen that urban water demand in Hefei shows a increasing tendency, indicating the contradiction between water supply and demand is more serious in the near future. Therefore, we should develop further water saving technology and save water regarding limited water resources in future.

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